Characterizing Revealing and Arbitrage-Free Financial Markets

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Characterizing revealing and arbitrage-free financial markets

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Abstract

Radner (Econometrica 47: 655-678, 1979) introduces a general equilibrium model of asymmetric information where "agents have a 'model' or 'expectations' of how equilibrium prices are determined". They would only infer private information of other agents from comparing actual prices and price forecasts with their theoretical values at a price revealing equilibrium. De Boisdeffre (Economic Theory Bulletin 4(1), 2016) shows that agents having private anticipations and no price model may still update their beliefs from observing trade on financial markets, until all arbitrage is precluded. The informational refinement consists in successively eliminating anticipations, which would grant an unlimited arbitrage, if realizable. Thus, agents simply observe, respond and learn from arbitrage opportunities on portfolios, as they would do on actual markets. This model is consistent with all kinds of assets and uncountably many forecasts. We now study markets, which preclude arbitrage, and show the information markets may convey depends on the span of asset payoffs in agents' commonly expected states. We provide conditions, under which markets are non informative, or, typically, partially or fully revealing.

Key words: anticipations, inferences, perfect foresight, rational expectations, financial markets, asymmetric information, arbitrage.

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1 Introduction

Asymmetrically informed agents may infer information from observing prices or trade volumes on markets. Thus, in the Radner (1979) rational expectation setting "*agents have a 'model' or 'expectations' of how equilibrium prices are determined*". They may infer private information of other agents from comparing actual prices and price expectations with their theoretical values at a price revealing equilibrium. Yet, equilibrium may fail to exist. Existence in Radner's model only holds generically.

Hereafter, we drop both Radner's (1972, 1979) classical assumptions that agents have rational expectations and a perfect foresight of future prices. Instead, we consider a two-period model with uncountably many states, also called anticipations, expectations or forecasts. The state space captures the exogenous uncertainty, stemming from nature's play over future events, and also, typically, an endogenous uncertainty, resulting from the fact that agents' characteristics, forecasts and beliefs may be private. Agents' forecasts form idiosyncratic subsets of the state space.

Assets of any kind may be exchanged at the first period, whose payoffs, at the second, are state dependent. Starting from their idiosyncratic sets of anticipations, agents may update their beliefs from observing prices or trade opportunities on portfolios, as in De Boisdeffre (2016, [3]). Namely, they may narrow down their expectation sets, in finitely many steps, by successively eliminating forecasts, that would grant them an unlimited arbitrage opportunity, if correct.

The current paper studies the payoff structure of arbitrage-free markets, and the information markets may reveal. It shows this information depends on the span of asset payoffs in agents' commonly expected states. It provides conditions, under which markets are non-revealing or, typically, partially or fully revealing. In particular, when agents have perfect foresight, dropping Radner's (1979) rational expectation assumption to deal with asymmetric information not only restores the full existence property of equilibrium, along De Boisdeffre (2007), but provides insights on the information agents reach at equilibrium.

The paper is organized as follows: Section 2 recalls the model and results of De Boisdeffre (2016, [3]). Section 3 studies the information markets may reveal and characterizes arbitrage-free financial structures, when agents' expectation sets are all finite. Section 4 generalizes the results to the general model.

2 The model

2.1 The information and financial structures

We consider a pure-exchange economy with two periods ($t \in \{0,1\}$) and finitely many agents, $i \in I := \{1, ..., m\}$, having uncertainty at the first period about which state, ω, will prevail tomorrow out of a state space, denoted by Ω. This set, Ω, stands for any relatively open subset with cardinality of the continuum of a metric space (e.g., $\Omega :=]0,1[$). States are also called forecasts, anticipations or expectations.

At $t = 0$, each agent, $i \in I$, receives a private information signal, in the form of a compact sub-set, Ω_i, of Ω, informing her that tomorrow's state will be in Ω_i. She then elects a consistent probabilty distribution on $(\Omega, \mathcal{B}(\Omega))$, called her belief, where $\mathcal{B}(\Omega)$ denotes the Borel sigma-algebra of Ω.

The information structure, (Ω_i), is given throughout, and such that $\underline{\mathbf{\Omega}} := \cap_{i \in I} \Omega_i$ is non-empty. It may be refined along the following Definition:

Definition 1 *A collection,* $(P_i) := (P_i)_{i \in I}$ *of compact subsets of* Ω *is said to be an anticipation structure, or structure, if:*

(a) $\cap_{i=1}^m P_i \neq \varnothing$.

Their set is denoted by $\mathcal{AS}$*. A structure,* $(P_i') \in \mathcal{AS}$*, is said to refine, or to be a refinement of* $(P_i) \in \mathcal{AS}$*, and we denote it by* $(P_i') \leq (P_i)$*, if:*

(b) $P_i' \subset P_i$, $\forall i \in I$.

A refinement, $(P_i') \in \mathcal{AS}$*, of* $(P_i) \in \mathcal{AS}$*, is said to be self-attainable if:*

(c) $\cap_{i=1}^m P_i' = \cap_{i=1}^m P_i$.

For every $\varepsilon > 0$*, every* $\overline{\omega} \in \Omega$ *and every probability distribution,* π*, on* $(\Omega, B(\Omega))$*, we let* $B(\omega, \varepsilon) := \{\omega \in \Omega : |\omega - \overline{\omega}| < \varepsilon\}$*, and* $P(\pi) := \{\omega \in \Omega : \pi(B(\omega, \varepsilon)) > 0, \forall \varepsilon > 0\}$ *be the support of* π*. The* m *probabilities,* (π_i)*, on* $(\Omega, B(\Omega))$*, are said to be a structure of beliefs if* $(P(\pi_i))$ *is an anticipation structure. Then,* (π_i) *is said to support* $(P(\pi_i)) \in \mathcal{AS}$*. Given* $(P_i) \in \mathcal{AS}$*, the set of stuctures of beliefs, which support* (P_i)*, is denoted by* $\Pi[(P_i)]$.

The above specification of information embeds, in particular, De Boisdeffre's (2007 and 2016, [4]), where information sets are, respectively, finite sets, and compact subsets of $\{1, ..., K\} \times \mathbb{R}^L_{++}$, for some integers K and L. In the latter, agents face an exogenous uncertainty amongst finitely many events, combined with an endogenous uncertainty on consumption prices in $\mathbb{R}^L_{++}$, stemming from the fact that individual characteristics, forecasts and beliefs are all private.

Agents exchange finitely many assets, $j \in \mathcal{J} := \{1, ..., J\}$, at $t = 0$, whose cash payoffs, $v_j(\omega) \in \mathbb{R}$, are conditional on the occurence of a state $\omega \in \Omega$, at $t = 1$, and define a row vector, $V(\omega) = (v_j(\omega)) \in \mathbb{R}^J$. The mapping $\omega \in \Omega \mapsto V(\omega)$ is assumed to be continuous. Agents' positions in assets define portfolios, $z \in \mathbb{R}^J$. Given an asset price, $q \in \mathbb{R}^J$, a portofolio, $z \in \mathbb{R}^J$, costs $q \cdot z$ units of account at $t = 0$, and promises $V(\omega) \cdot z$ units tomorrow, in each state, $\omega \in \Omega$, if ω obtains. We now present arbitrage.

Definition 2 *A price, $q \in \mathbb{R}^J$, is said to be a common no-arbitrage price of a structure, $(P_i) \in \mathcal{AS}$, or the structure (P_i) to be q-arbitrage-free, if the following condition holds:*
(a) $\nexists(i,z) \in I \times \mathbb{R}^J : -q \cdot z \geqslant 0$ and $V(\omega) \cdot z \geqslant 0$, $\forall \omega \in P_i$, with one strict inequality;
A structure, which admits a common no-arbitrage price, is called arbitrage-free.

Claim 1 recalls a characterization of common no-arbitrage prices and structures.

Claim 1 *Let $(P_i) \in \mathcal{AS}$, $(\pi_i) \in \Pi[(P_i)]$ and $q \in \mathbb{R}^J$ be given, along Definition 1. For each $i \in I$, we denote by $L_2^{++}(\pi_i)$ the set of mappings, $f : P_i \to \mathbb{R}$, in the Riesz space $L_2(\pi_i)$, such that $f(\omega) > 0$ π_i-almost surely. The following statements are equivalent:*
(i) $q \in Q_c[(P_i)]$, that is, (P_i) is q-arbitrage free;
(ii) $\forall i \in I$, $\exists f_i \in L_2^{++}(\pi_i)$, such that $q = \int_{\omega \in P_i} V(\omega) f_i(\omega) d\pi_i(\omega)$;
Besides, (P_i) is arbitrage-free if and only if it meets the following AFAO Condition:
(iii) $\nexists(z_i) \in (\mathbb{R}^J)^m : \sum_{i=1}^m z_i = 0$, $V(\omega_i) \cdot z_i \geqslant 0$, $\forall (i, \omega_i) \in I \times P_i$, with at least one strict.

Proof The proof is given under Claim 2 in De Boisdeffre (2016, [3]). □

2.2 Informational properties

As long as arbitrage opportunities remain, agents cannot agree on assessing prices. Yet, they may exchange portfolios and learn from trade, along Claim 2. Let:

- $A_i^1 = \varnothing$ and $\Omega_i^1 := \Omega_i$, for each $i \in I$;
- with A_i^n and Ω_i^n defined at step $n \in \mathbb{N}$, for each $i \in I$, we let, for each $i' \in I$:

$$A_{i'}^{n+1} := \{\overline{\omega} \in \Omega_{i'}^n : \exists (z_i) \in (\mathbb{R}^J)^m,\ \sum_{i=1}^m z_i{=}0,\ V(\overline{\omega}){\cdot}z_{i'}{>}0,\ V(\omega_i){\cdot}z_i{\geqslant}0,\ \forall (i, \omega_i) \in I{\times}\Omega_i^n\}$$

$\Omega_{i'}^{n+1} := \Omega_{i'}^n \backslash A_{i'}^{n+1}$, i.e., agents rule out expectations, granting an arbitrage.

Claim 2 *The sequence, $\{(\Omega_i^n)\}_{n \in \mathbb{N}}$, satisfies the following Assertions:*
(i) there exists one coarsest arbitrage-free self-attainable refinement, (Ω_i^*), of (Ω_i);
(ii) $\exists N \in \mathbb{N} : \forall n > N, \forall i \in I$, $\Omega_i^n = \Omega_i^*$.

Proof The proof is given under Claim 4 in De Boisdeffre (2016, [3]). □

The above inferences, which require no price model, would be made on actual financial markets, by agents having incomplete information and operating via private trade houses, competing to make profits. When applied to the De Boisdeffre (2016, [4]) model, agents having inferred (Ω_i^*) can always reach equilibrium - shown to exist whatever the beliefs - if their common anticipations embed a so-called 'minimum uncertainty set', which features an incompressible uncertainty on tomorrow's commodity prices, stemming from agents' private beliefs. It is, therefore, useful to draw informational implications of agents' inferences and financial strutures. Sections 3 and 4 address these issues, successively in the finite and the general cases.

3 Information markets may reveal with finite anticipations

To simplify exposition, anticipation sets, Ω_i (for each $i \in I$), are, at first, finite. We le $(\Omega_i^*) \leqslant (\Omega_i)$ be the coarse arbitrage-free refinement of Claim 2, $S := \cup_{i \in I} \Omega_i$, $S^* := \cup_{i \in I} \Omega_i^*$ and $\underline{\mathbf{\Omega}} = \cap_{i \in I} \Omega_i$. Individual state prices replace mappings in Claim 1-(ii). We define (for some $J^* \leqslant J$, with a slight abuse in notations) the $S \times J$ and $S^* \times J^*$ matrixes, $V := (V(\omega)) := (v_j(\omega))_{j \in \{1,...,J\}, \omega \in S}$ and $V^* := (V^*(\omega)) := (v_j(\omega))_{j \in \{1,...,J^*\}, \omega \in S^*}$, from the payoff mapping of Section 2, by costlessly eliminating redundant assets, and:

- $Z_\omega := \{ \ z \in \mathbb{R}^J : \ V(\omega) \cdot z = 0 \ \}$, for each $\omega \in S$, and $Z_\omega^\perp$ its orthogonal;
- $Z_\omega^* := \{ \ z \in \mathbb{R}^{J^*} : \ V^*(\omega) \cdot z = 0 \ \}$, for each $\omega \in S^*$, and $Z_\omega^{*\perp}$ its orthogonal;
- $Z_i := \cap_{\omega \in \mathbf{\Omega}_i} Z_\omega$, $Z = \sum_{i \in \mathbf{I}} Z_i$, for each $i \in I$, and their orthogonals, $Z_i^\perp$, $Z^\perp = \cap_{i \in \mathbf{I}} Z_i^\perp$;
- $\underline{Z} = \cap_{\omega \in \underline{\mathbf{\Omega}}} Z_\omega$, $\underline{Z}^* = \cap_{\omega \in \underline{\mathbf{\Omega}}} Z_\omega^*$, and their orthogonals, $\underline{Z}^\perp$ and $\underline{Z}^{*\perp}$;
- $Z_i^* := \cap_{\omega \in \mathbf{\Omega}_i^*} Z_\omega^*$, , $Z^* = \sum_{i \in \mathbf{I}} Z_i^*$ and their orthogonals, $Z_i^{*\perp}$ and $Z^{*\perp} = \cap_{\omega \in \mathbf{\Omega}_i^*} Z_i^{*\perp}$;

- W_ω, the straight line of $\mathbb{R}^J$, generated by $w_\omega := (v_j(\omega))_{j\in\{1,\ldots,J\}}$, for each $\omega \in S$;

- $\underline{W} := \sum_{\omega\in\mathbf{\Omega}} W_\omega$ and their orthogonals, $W_\omega^\perp$ and $\underline{W}^\perp$.

- We similarly define W_ω^*, w_ω^*, for each $\omega \in S^*$, $\underline{W}^*$ and orthogonals.

Claim 3 *The above vector spaces meet the following Assertions:*

(i) $W_\omega = Z_\omega^\perp$*,* $\forall\omega \in S$*, and* $W_\omega^* = Z_\omega^{*\perp}$*,* $\forall\omega \in S^*$*;*

(ii) $\mathbb{R}^J = \sum_{i\in\mathbf{I}} Z_i^\perp$ *and* $\mathbb{R}^{J^*} = \sum_{i\in\mathbf{I}} Z_i^{*\perp}$*;*

(iii) $Z \subset \underline{Z} = \underline{W}^\perp$ *and* $Z^* \subset \underline{Z}^* = \underline{W}^{*\perp}$*;*

(iv) $(\Omega_i^*) = (\Omega_i)$ *if and only if the following condition holds:*

(I) $\nexists (z_i) \in (\mathbb{R}^J)^m : \sum_{i=1}^m z_i = 0,\ V(\omega_i)\cdot z_i \geqslant 0,\ \forall (i,\omega_i) \in I \times \Omega_i$*, with at least one strict*;

(v) If $\underline{Z} = \{0\}$*, then,* $(\Omega_i^*) = (\Omega_i)$*, i.e.,* (Ω_i) *is non-revealing (or arbitrage-free);*

(vi) If $I \neq I' = \{i \in I : \Omega_i \neq \mathbf{\Omega}\}$*,* $(\Omega_i^*) = (\Omega_i)$ *if and only if the below condition holds:*

(II): $\nexists (i,z) \in I' \times \underline{Z} : V(\omega)\cdot z \geqslant 0,\ \forall\omega \in \Omega_i$*, with at least one strict inequality;*

(vii) Assume that $I \neq I'$*,* $\underline{Z} \neq \{0\}$ *and, costlessly for some* $J_1 \leqslant J$*, that the first* J_1^{th} *assets yield a Hamel basis of* $\underline{Z}$*. If* $\{v_j(\omega)\}_{j\in\{1,\ldots,J_1\},\omega\in S\setminus\mathbf{\Omega}} \subset \mathbb{R}_+$*, then,* $Z^* = \underline{Z}^* = \{0\}$ *and, moreover,* (Ω_i) *is fully-revealing if* $\{0\} \neq \{v_j(\omega)\}_{j\in\{1,\ldots,J_1\}}$*, for each* $\omega \in S\setminus\mathbf{\Omega}$*.*

Proof *(i)* Let $\omega \in S$ be given. If $w_\omega = 0$, then $W_\omega = Z_\omega^\perp = \{0\}$. If $w_\omega \neq 0$, the spaces, W_ω and $Z_\omega^\perp$, are 1-dimensional and contain w_ω, i.e., coincide. The rest is alike. □

(ii) The relations $(\sum_{i\in\mathbf{I}} Z_i^\perp)^\perp = \cap_{i\in\mathbf{I}} Z_i = \{0\}$ and $(\sum_{i\in\mathbf{I}} Z_i^{*\perp})^\perp = \{0\}$ hold, from the elimination of redundant assets, hence, $\mathbb{R}^J = \sum_{i\in\mathbf{I}} Z_i^\perp$ and $\mathbb{R}^{J^*} = \sum_{i\in\mathbf{I}} Z_i^{*\perp}$ hold. □

(iii) From the above definitions and Assertion *(i)*, the relations $Z^\perp = (\sum_{i\in\mathbf{I}} Z_i)^\perp = \cap_{i\in I} Z_i^\perp = \cap_{i\in I}(\sum_{\omega\in\mathbf{\Omega}_i} Z_\omega^\perp) = \cap_{i\in I}(\sum_{\omega\in\mathbf{\Omega}_i} W_\omega) \supset \underline{W} = \underline{Z}^\perp$ hold. Assertion *(iii)* follows. □

(iv) Assertion *(iv)* states the AFAO characterization of Claim 1, above, in the finite dimensional case, proved directly in Cornet-De Boisdeffre (2002, p. 401). □

(v) Assume that $\underline{Z} = \{0\}$ and, by contraposition, that (Ω_i) fails to be arbitrage-free. From Assertion (iv), there exists $(z_i) \in (\mathbb{R}^J)^I \backslash \{0\}$, such that $\sum_{i=1}^{m} z_i = 0$ and $V(\omega) \cdot z_i \geqslant 0$ for every pair $(i, \omega) \in I \times \underline{\mathbf{\Omega}}$. These joint relations imply $V(\omega) \cdot z_i = 0$ for every $(i, \omega) \in I \times \underline{\mathbf{\Omega}}$, that is, $(z_i) \in \underline{Z}^m \backslash \{0\}$, contradicting the fact that $\underline{Z} = \{0\}$. This contradiction proves that (Ω_i) is arbitrage-free, i.e., from Claim 3, $(\Omega_i^*) = (\Omega_i)$. □

(vi) Assume, by contraposition, that $I \neq I'$, $(\Omega_i^*) = (\Omega_i)$ and Condition (II) of Assertion (vi) fails. Then, there exists $(i, z) \in I' \times \underline{Z}$, such that $V(\omega) \cdot z \geqslant 0$ for all $\omega \in \Omega_i$ and $\sum_{\omega \in \Omega_i} V(\omega) \cdot z > 0$. One agent, say $j \in I \backslash I'$ is fully informed. Then, Condition (I) of Assertion (iv) fails with $(z_i, z_j) = (z, -z)$, that is, (Ω_i) fails to be arbitrage-free, which contradicts the above relation, $(\Omega_i^*) = (\Omega_i)$. This contradiction shows the relation $(\Omega_i^*) = (\Omega_i)$ implies Condition (II) to hold. Assume, now, that $(\Omega_i^*) \neq (\Omega_i)$. From Assertion (iv) and the proof of Assertion (v), there exists $(z_i) \in (\underline{Z})^m$, such that $V(\omega_i) \cdot z_i \geqslant 0$ for each $(i, \omega_i) \in I \times \Omega_i$, with one strict inequality, hence, Condition (II) fails. This proves that Condition (II) implies the relation $(\Omega_i^*) = (\Omega_i)$ to hold. □

(vii) Assertion (vii) stems from Assertion (vi) and redundant asset elimination. □

Claim 3 shows that the information markets may reveal depends on the span of assets' payoffs in commonly expected states. Thus, if $\underline{W} = \underline{Z}^{\perp} = \mathbb{R}^J$, financial markets are non-revealing. In economies where real assets are exchanged and agents have many common forecasts (including price forecasts), markets are, thus, typically non-informative (with $\underline{W} = \mathbb{R}^J$). Contrarily, financial markets insuring primarily idiosyncratic risks (with $\underline{W} \neq \mathbb{R}^J$), would typically be fully revealing, if one agent has full information (along Claim 4-(vii)), or partially revealing otherwise.

We now generalize Claim 3 to the general setting.

4 Information markets may reveal in the general model

Studying markets' informational properties is eased by the fact that all vector spaces, define below, are finite, hence, have orthogonal supplements. As above, we let $S := \cup_{i\in I}\Omega_i$, $S^* := \cup_{i\in I}\Omega_i^*$ and derive the mappings $V : \omega \in S \mapsto V(\omega) := (v_j(\omega))_{j\in\{1,...,J\},\omega\in S}$ and V^*: $\omega \in S^* \mapsto V(\omega) := (v_j(\omega))_{j\in\{1,...,J^*\}}$, from the one in Section 2, where we obtain J and $J^* \leqslant J$ by eliminating redundant assets, if any.

For each agent, $i \in I$, and every state state, ω, in S or S^*, we define, in the general model, the vector spaces, Z_ω, Z_ω^*, W_ω, W_ω^*, Z_i, Z_i^*, Z, $\underline{Z}$, Z^*, $\underline{Z}^*$ and their orthogonals, in the same way as in Section 3 for the finite economy. We define the vector spaces, $\underline{W} :=$ $\{ z \in \mathbb{R}^{J^*} : \exists f \in L_2(\pi)$, *along Claim 1, such that* $z = \int_{\omega\in\underline{\mathbf{\Omega}}} V(\omega)f(\omega)d\pi(\omega)$ $\}$ and, similarly, $\underline{W}^*$, for any belief, π, with support $\underline{\mathbf{\Omega}}$, and their orthogonals, $\underline{W}^\perp$ and $\underline{W}^{*\perp}$.

Claim 4 states the properties of Claim 3, above, for the general model.

Claim 4 *The above vector spaces meet the following Assertions:*

(i) $W_\omega = Z_\omega^\perp$, $\forall\omega \in S$, *and* $W_\omega^* = Z_\omega^{*\perp}$, $\forall\omega \in S^*$;

(ii) $\mathbb{R}^J = \sum_{i\in\mathbf{I}} Z_i^\perp$ *and* $\mathbb{R}^{J^*} = \sum_{i\in\mathbf{I}} Z_i^{*\perp}$;

(iii) $Z \subset \underline{Z} = \underline{W}^\perp$ *and* $Z^* \subset \underline{Z}^* = \underline{W}^{*\perp}$;

(iv) $(\Omega_i^*) = (\Omega_i)$ *if and only if the following condition holds:*

(I) $\nexists(z_i) \in (\mathbb{R}^J)^m : \sum_{i=1}^m z_i = 0$, $V(\omega_i)\cdot z_i \geqslant 0$, $\forall(i,\omega_i) \in I\times\Omega_i$, *with at least one strict;*

(vi) If $I \neq I' = \{i \in I : \Omega_i \neq \underline{\mathbf{\Omega}}\}$, $(\Omega_i^*) = (\Omega_i)$ *if and only if the below condition holds:*

(II): $\nexists(i,z) \in I'\times\underline{Z} : V(\omega)\cdot z \geqslant 0$, $\forall\omega \in \Omega_i$, *with at least one strict inequality;*

(vii) Assume that $I \neq I'$, $\underline{Z} \neq \{0\}$ *and, costlessly for some* $J_1 \leqslant J$, *that the first* J_1^{th} *assets yield a Hamel basis of* $\underline{Z}$. *If* $\{v_j(\omega)\}_{j\in\{1,...,J_1\},\omega\in S\backslash\underline{\mathbf{\Omega}}} \subset \mathbb{R}_+$, *then,* $Z^* = \underline{Z}^* = \{0\}$ *and, moreover,* (Ω_i) *is fully-revealing if* $\{0\} \neq \{v_j(\omega)\}_{j\in\{1,...,J_1\}}$, *for each* $\omega \in S\backslash\underline{\mathbf{\Omega}}$.

Proof The proof of Claim 4 is similar to that of Claim 3 and left to the reader. Similar arguments apply, due, in particular, to the fact that all above vector spaces have a finite Hamel basis, in either sets $\{V(\omega)\}_{\omega \in S}$ or $\{V^*(\omega)\}_{\omega \in S^*}$. □

Claims 3 and 4 state simple characterizations of arbitrage-free structures. In economies where real assets may be traded and the state space embeds an endogenous uncertainty on prices, such as De Boisdeffre's (2016, [4]), markets would be non-revealing (with $\underline{W} = \mathbb{R}^J$), because the set of common forecasts, $\underline{\boldsymbol{\Omega}}$, and the span of payoffs are typically large. In the latter economy, equilibrium exists if the set $\underline{\boldsymbol{\Omega}}$ includes a so-called minimum incertainty set, Δ, which features the uncertainty agents could face on prices, because their forecasts and beliefs are private. This set, Δ, embeds, in particular, all stantard perfect foresight equilibrium prices. Studying its cardinality, would provide additional insights on the information markets convey.

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