Insurance pools for new and undiversifiable risk

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Preliminary version

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Abstract

This paper discusses the decision of the European Commission not to renew the antitrust exemption for the setting up of syndicates in the insurance industry. Pools are constituted to provide insurance for undiversifiable and/or new risks for which insurers with private expertise are capacity constrained. Our objective is to study if such syndicates improve insurance supply. Organizing this supply amounts to sharing a common value divisible good between capacity constrained and privately informed agents with a reserve price. Pools turn out to operate as a uniform price auction with an "exit/re-entry" option that we compare to a discriminatory auction where no specific agreements are needed. Both auction formats lead to different coverage/premium tradeoffs. If at least one insurer provides an optimistic expertise, the pool offers both lower premiums and higher coverage. This result is reversed when all insurers are pessimistic about the risk. Static comparative results with respect to capacity constraints and reserve price are provided.

Keywords: pool insurance, competition, undiversifiable risk

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1 Introduction

On December 13, 2016 the European Commission took note of the expiry of the Insurance Block Exemption Regulation (IBER) on March 31, 2017. The IBER that was established in 1992 (and regularly renewed) authorized certain categories of agreements, decisions and concerted practices in the insurance sector "*to ensure the proper functioning of this sector and promote consumer interest*" [1]. The application we focus on is the practice of insurance companies to co-operate and jointly insure risks via 'co-(re)insurance pools'. These co-(re)insurance pools are defined as *(...) groups set up by insurance undertakings either directly or through brokers or authorized agents (...) whereby a certain part of a given risk is covered by a lead insurer and the remaining part of the risk is covered by follow insurers who are invited to cover that remainder.*"

The decision to maintain pools or not not has been discussed for several years. This discussion was centered on the efficiency of such pools. What is the impact of this form of cooperation on the supply of insurance and on the pricing of insurance policies? To what extent is the performance of the insurance industry affected? Would we observe a decrease in insurance capacity and thus less coverage for new risks without pools? According to insurers, pools for nuclear risks (Assuratome) or environmental risks (Assurpol) are the only solution to provide insurance. Also, they argue that pools enable insurance companies to share knowledge and experience about certain less frequently occurring risks, which should, according to them, benefit to both sides of the market. After a large consultation, the European Commission decided not to renew the exemption for pools, as it fears that pools generate a restriction of competition.[2] The new policy is to provide a case by case analysis on the principle of self-assessment. The objective of the paper is to analyze the efficiency of such pools and other possible agreements to discuss the European Commission decision.

Pools are usually constituted of a small number of firms and all risks are not eligible to

[1](REPORT FROM THE COMMISSION TO THE EUROPEAN PARLIAMENT AND THE COUNCIL On the functioning of Commission Regulation (EU) No 267/2010 on the application of Article 101(3) of the Treaty on the functioning of the European Union to certain categories of agreements, decisions and concerted practices in the insurance sector)

[2]Arguments of the European Commission decision can be found in the report "REPORT FROM THE COMMISSION TO THE EUROPEAN PARLIAMENT AND THE COUNCIL On the functioning of Commission Regulation (EU) No 267/2010 on the application of Article 101(3) of the Treaty on the functioning of the European Union to certain categories of agreements, decisions and concerted practices in the insurance sector".

this exemption. These pools apply to new[3] and/or undiversifiable risks. The insurance of known risks is usually possible by standard diversification principles. When risks cannot be washed out by diversification (these risks are called non-diversifiable risks), insurance economics theory teaches us that co-(re)insurance pools allow risk sharing between the members (see for instance Doherty and Dionne, 1993). Pools "are necessary to allocate risk efficiently in the economy and, under some conditions, they create social value by disseminating undiversifiable risks within the largest possible set of risk-bearers" as claimed by Bobtcheff et al. (2015). However, when contemplating insuring new risks, insurers use their own expertise to evaluate them. At the industry level, this creates informational asymmetries not only between insurers and insurees, but also between insurers. The determination of the characteristics of insurance products is therefore much more complicated. Also, the undiversifiable nature of the risks may generate a lack of sufficient insurance capacity. These capacity constraints may come either from legal solvency regulation constraints and capital requisites that are imposed to prevent from insurers' bankruptcy (Solvency II) or from the characteristics of the risk itself ("*extraordinary large risks that occur in irregular intervals but may lead to very large damage claims*" for instance.

We propose to develop a unified theoretical model to analyse different pooling agreements as well as alternative forms of insurance cooperations. Different bargaining rules between insurers will be considered depending on whether a leader exists or not. This leads us to study both existing pool agreements and alternative organizations. This theoretical model is a simplified representation of insurers' interactions based on empirical findings of Ernst and Young report (2014) and takes into account the legal rules prevailing in the insurance industry. Ernst and Young (2014) provide a detailed description of the procedures leading to pool agreements in several European countries. Even if some country-specific differences exist, they share some common features. First, pools are generally constituted within a two-round auction, which defines a leading insurer and following ones. The leader's selection process may combine the following factors: capacity, premium, insurer's expertise or reputation, terms and conditions of the offer. As Ernst and Young (2014) note, "the followers are usually invited to either accept or decline or take a share of the risk on the same terms and conditions as the lead insurer". Based

[3] Defined in the Regulation as "risks which did not exists before, and for which insurance cover requires the development of an entirely new insurance product not involving an extension, improvement or replacement of an existing insurance product.

on this, we propose to study a simplified scenario in which the leader is selected on the basis of a single factor to disentangle the nature of the strategic interaction on the market performance. This analysis aims at understanding on which factor (price, capacity, or expertise) a pool should be constituted depending on the objectives of the decision-maker.

Organizing insurance supply then amounts to sharing a common value divisible good between capacity constrained agents with a reserve price where agents have private information. The question of agreeing on a common coverage of a risk is indeed akin to the one of exchanging Treasury debt and other divisible securities. These bonds are usually exchanged through a uniform auction in which a broker selects a price that equalizes the supply and demand for bonds or through a discriminatory auction. Not only are the pool's rules specific to the insurance industry, but also the nature of the good (the risk) that is exchanged (reserve price and capacity constraints) so that there is a need to develop a theoretical model.

The model we consider builds on the following elements. There exist an exogenous demand for insurance : insurees ask for a coverage for which they have a reserve premium. Insurers are capacity constrained and a receive a private signal about the common value risk. They syndicate through a particular two-round auction. The game we consider is a uniform price auction with an exit/re-entry option. We characterize the equilibrium risk premium of this game and the resulting insurance capacity offered. We then consider an a discriminatory auction in which each insurer offers its own conditions (capacity and premium). This scenario fits the situation where a broker meets each insurer individually so that no specific agreement between insurers is needed. This leads us to compare different auction pricing rules and to understand the role of the re-entry option. All auction formats are compared with respect to premiums and coverage taking into account the impact of different markets and characteristics (intensity of competition, risk aversion, affiliation).

We provide two kind of results: some results directly help to the discussion of the European Commission and other complete the auction literature.

Let us first discuss the outcome of the pool. We determine the unique equilibrium in symmetric and strictly increasing bidding strategies. Conditional on bidding, the equilibrium strategy completely reveals the signal an insurer observed. The reserve price implies the existence of a maximum signal determining the participation to the first round of the auction. The equilibrium exhibits both a complete market failure (no insurance) when both insurers have private pessimistic evaluations (above the threshold) and a partial market failure (partial insurance) when only one insurer is pessimistic about the risk. The re-entry option impacts these market failures in two ways: insurers refrain from bidding in the first round (increasing the no-insurance region) but an insurer always has the possibility to re-enter to auction in the second round if he discovers that his opponent received a good signal (increasing the full insurance region). All these market failure regions are affected by the parameters of the model: intensity of capacity constraints, reserve price and degree of common information on the risk. In particular, we show that new risks (known risks) lead to a more (lower) complete market failure but the partial market failure reduces (increases).

Considering the discriminatory auction, the equilibrium is semi-pooling or separating and also involves some complete and partial market failure regions. We show that both auction formats lead to different coverage/premium tradeoffs. We compare these two auctions for any possible realization of the two insurers' signals. If at least one insurer provides an optimistic expertise about the risk, the pool offers both lower premiums and higher coverage. If all insurers receive pessimistic information, the discriminatory auction offers a better coverage at a lower leader's premium but at a higher follower's premium.

The other contribution of this paper is to complement the auction literature on discriminatory auction of a common value divisible good with a reserve price. We determine the type of equilibrium that exists (separating or semi-pooling) in a bi-dimensional setting (reserve price and strength of competition).

From now, the literature on undiversifiable risks has focused on the risk sharing problem between insurers and policyholders. This risk sharing problem is analyzed for instance in Doherty and Dionne (1993) or Mahul and Wright (2003). Doherty and Dionne (1993) introduce a new form of insurance contract called Decomposed Risk Transfer contract (DRT contract) defined by an insurance policy packaged with a residual claim on the insurance pool. They show that this contract increases policyholders welfare. They characterize the optimal coverage and the risk premium as a function of the cost of risk bearing derived from asset pricing models. Our setting builds on such a two dimensional contract (a risk premium and a coverage). We do not discuss any risk sharing issue associated with the undiversifiable risk, but consider that such risk implies legal capacity constraints for insurers. In our model, the pool risk premium (paid by the policyholders) may differ from the actuarial rate (paid by the insurer) because of the particular competition emerging from the pool. Our main objective is to analyze how to organize insurance supply (premiums and coverage).

The paper is organized as follows. We present the model in section 2. We then solve the equilibrium of the pool in section 3. In section 4, we introduce an alternative organization. Section 5 is devoted to the comparison between the two auction formats. All proofs are relegated to the appendix.

2 The model

Two identical insurers are asked for the coverage of an undiversifiable risk where non diversifiability translates into capacity constraints.

2.1 Risk, insurers and contract

A risk averse agent is exposed to an undiversifiable risk characterized by a loss of size L occurring with probability p. This agent asks for an exogenous coverage βL. An insurance contract is completely defined by the unit risk premium P.[4] The expected utility with such a contract writes

$$V(P,p;\beta) = pu\left(w - L + \beta L - \beta LP\right) + (1-p)u\left(w - \beta LP\right)$$

where u denotes the increasing and concave utility function, w the agent's initial wealth, βL is the indemnity paid by insurers in case of loss, and βLP is the insurance premium paid for this coverage. We define $\overline{P}(p) \equiv \overline{P}$ as the maximum premium that the agent is willing to pay for this coverage. At $\overline{P}$, he is indifferent between insuring or not $V(\overline{P},p;\beta) = V(0,p;0)$.

The two identical risk neutral insurers, i and j, compete for the coverage of this risk by choosing the price at which they provide insurance and the quantity they insure. The minimum risk premium that they are willing to accept for this coverage is the actuarial premium rate p so that there are gains from trade if and only if $p \leq P \leq \overline{P}$. We then obtain that insurers' net expected benefit of such a contract is linear with respect to β and writes

$$\beta L\left(P - p\right).$$

[4]We assume that P is independent of β.

The existence of solvency regulation and capital requisites for the coverage of such new and/or undiversifiable risk implies that insurers are capacity constrained. A single insurer can not offer more than a proportion $\overline{\beta}_i \leq \beta$, with $\overline{\beta}_i = \overline{\beta}_j = \overline{\beta}$. As a consequence, if gains from trade exist, an insurer prefers to insure its maximum capacity $\overline{\beta}$.

We also assume that the market is too small to absorb the full capacity of the two insurers, i.e. $\beta \leq 2\overline{\beta}$.[5] To measure the strength of competition on the insurance market, we define

$$\kappa = \frac{2\overline{\beta} - \beta}{\overline{\beta}} \in [0, 1], \tag{1}$$

which can be interpreted as the relative excess supply. When $\kappa = 0$ ($\overline{\beta} = \beta/2$), the two insurance companies may sell their entire capacity so that there is no competition. On the contrary, when $\kappa = 1$ ($\overline{\beta} = \beta$), a unique insurer could satisfy the whole demand leading to intense competition. Note that equation (1) implies that the proportion of the total demand an insurer can satisfy by itself, $\overline{\beta}/\beta$, equals $1/(2 - \kappa)$.

2.2 Insurers' expertise

We assume that the probability of the undiversifiable risk p is not perfectly known by the agents. All agents have the same prior on this probability, denoted $p_0 = \mathbb{E}[p]$, the a priori actuarial premium. This belief defines the *a priori* maximum premium $\overline{P}_0$ implicitly defined by $V(\overline{P}_0, p_0; \beta) = V(0, p_0; 0)$.

Insurers are assumed to be expert in the evaluation of such risks. They have the ability to better identify the true risk. This assumption reflects the fact that insurers often concentrate their activities in specific lines of business and can use their expertise to infer the probability of these risks. As a consequence, we assume that insurers can obtain a costless signal related to the true probability. S_i (resp. S_j) is the signal privately observed by insurer i (resp. j). As a consequence, the actuarial premium rate is a function of insurers' private information. It is assumed to be the same for the two insurers and to be a symmetric function of all insurers' signals.

$$p(s_i, s_j) = p(s_j, s_i) \equiv \mathbb{E}\left[p | S_i = s_i, S_j = s_j\right]. \tag{2}$$

We impose the following regularity assumptions on the actuarial premium rate.

[5] Assuming sufficiently large capacities can avoid a monopoly outcome and allows to focus on the most interesting cases.

Assumption 1 *The actuarial premium rate p satisfies the following properties.*

(i) *Function p is twice continuously differentiable and strictly increasing in the two variables;*

(ii) $\mathbb{E}\left[p(S_i, 0)\right] < \overline{P}_0 < \mathbb{E}\left[p(S_i, 1)\right]$.

Observe that a high value of s signals a risk that is assumed to be more costly to insure and that some risks cannot be insured. Assumption 1(ii) means that if insurer j observes the best (resp. worst) possible signal, covering the risk is always (resp. never) profitable for insurer i.

The two signals S_i and S_j and p are assumed to be affiliated. Signals are distributed according to the same continuous distribution on the interval $[0, 1]$. Let $g(.|s)$ denote the (symmetric) probability distribution function of an insurer's signal conditional on the other insurer having observed signal s. Affiliation translates into the following assumption on the family of densities $g(.|s)$.

Assumption 2

$$\forall s_i' > s_i \text{ and } s_j' > s_j, \frac{g(s_i'|s_j')}{g(s_i'|s_j)} \geq \frac{g(s_i|s_j')}{g(s_i|s_j)}. \tag{3}$$

Let us also define signal $\widetilde{\sigma}$ as the maximal signal for which the two insurance companies accept to cover the risk in case they observe the same signal and function α that can be interpreted as an isocost curve evaluated at the maximal premium $\overline{P}_0$.[6]

Definition 1

(i) $\widetilde{\sigma}$ *is implicitly defined by*

$$p\left(\widetilde{\sigma}, \widetilde{\sigma}\right) = \overline{P}_0. \tag{4}$$

(ii) α *is implicitly defined by*

$$p(\alpha(x), x) = \overline{P}_0 \ \forall x \in [0, 1]. \tag{5}$$

Given our assumptions, if the premium is P, insurer i's net expected benefit of providing one unit of coverage writes[7]

$$P - \mathbb{E}\left[p(s_i, S_j)\right]. \tag{6}$$

[6] According to Assumption 1(ii), α is a decreasing function. Moreover, the symmetry of α with respect to its arguments implies that $\alpha^{-1} = \alpha$.

[7] In what follows, we normalize L to 1.

2.3 Insurers' syndication

The organization of insurance supply then amounts to the problem of sharing a common value divisible good between capacity constrained agents with a reserve price. Typically, this issue has been addressed for the particular case of Treasury Bonds. These bonds are usually exchanged through a uniform auction or a discriminatory auction. The insurance industry has its own practices to provide coverage for undiversifiable risks under capacity constraints. As we describe in the introduction, such arrangements are named co(re)insurance pools or co(re)insurance agreements.

The objective of this paper is to analyze different auction rules to constitute the syndicate, namely the pool (or the co(re)insurance agreements) and the more standard discriminatory auction. Each auction determines a game of incomplete information among the insurers: we look for a symmetric Bayesian Nash equilibrium that is increasing in the bidding strategies of each resulting game.

3 Analysis of the pool

3.1 Description of the pool

In this section, we model the most representative organization of the insurance sector, namely co(re)insurance pools and co(re)insurance agreements. We will refer to this representative organization as the "pool". Ernst and Young (2014) provides a detailed description of the procedures leading to agreements in several European countries. Even if some country-specific differences exist, they share some common features that we decide to highlight. The pool premium is unique and equals the lowest bid: this is a uniform auction. Ernst and Young (2014) also notes that "the followers are usually invited to either accept or decline or take a share of the risk on the same terms and conditions as the lead insurer". We summarize these features with the following rules.

- Each insurer performs a risk analysis and receives a private signal s_i.
- A first price auction determines the pool risk premium.
- If at least one insurer submits a bid $P_i \leq \overline{P}_0$,
  - o The insurer that submitted the smallest risk premium is the pool leader and sells $\overline{\beta}$ at price P^P

o The other insurer is the follower and observes P^P. It decides whether it sells $\beta - \overline{\beta}$ at price P^P or not

- If no insurer submits a bid, there is no trade.

This particular syndication works as if there exists two rounds. The rules state that the follower can join or quit the pool whatever its initial choice to submit a bid. A player submits a bid only if it is sufficiently optimistic about the risk ex ante. If it turns out to be the follower, it always has the possibility to quit the pool after having observed P^P. Also, if a player is too pessimistic to submit a bid ex ante, it may still, in a second round, re-enter and participate if the leader's bid reveals a good risk.

3.2 Separating equilibrium

We first look for an equilibrium in strictly increasing and symmetric biding strategies that are characterized by a threshold $\widehat{\sigma}^P$ such that

- when $s_i \leq \widehat{\sigma}^P$, firm i bids according to a strictly increasing bidding strategy $P^P(s_i)$ with $P^P(\widehat{\sigma}^P) = \overline{P}_0$,
- when $s_i > \widehat{\sigma}^P$, firm i is willing to participate in the second round only.

In such a separating equilibrium, the bid an insurer submits unambiguously reveals the signal it observes. The profit of firm i that observed a signal s_i and bids a risk premium $P^P(s_i)$ reads

$$\Pi^P(s_i) = \begin{cases} \overline{\beta}\left(1 - G(s_i|s_i)\right)\mathbb{E}\left[P^P(s_i) - p(s_i, S_j)|S_j > s_i\right] & \\ \quad + \left(\beta - \overline{\beta}\right)G(s_i|s_i)\,\mathbb{E}\left[\left(P^P(S_j) - p(s_i, S_j)\right)_+ |S_j < s_i\right] & \text{for } s_i \leq \widehat{\sigma}^P \quad (7a) \\ \left(\beta - \overline{\beta}\right)G\left(\widehat{\sigma}^P|\widehat{\sigma}^P\right)\mathbb{E}\left[\left(P^P(S_j) - p(s_i, S_j)\right)_+ |S_j < \widehat{\sigma}^P\right] & \text{for } s_i > \widehat{\sigma}^P \quad (7b) \end{cases}$$

Three terms compose the expression of $\Pi^P(s_i)$.

- The first term of equation (7a) corresponds to the case where insurer i observes the lowest signal. This happens when $S_j > s_i$, an event of probability $1 - G(s_i|s_i)$. In such case, firm i proposes the lowest risk premium and becomes the pool leader. It therefore serves $\overline{\beta}$ at its proposed price $P^P(s_i)$. We refer to this term as *the leader's value of firm i*.

- The second term of equation (7a) corresponds to the case where insurer i observes the highest signal. This happens when $S_j < s_i$, an event of probability $G\left(s_i|s_i\right)$. In such case, firm i proposes the highest risk premium and becomes the pool follower. It serves $\beta - \overline{\beta}$ at firm j's price $P^P(s_j)$. Note that if firm i's payoff turns out to be negative, it can withdraw from the pool (hence the subscript "$_+$"). Therefore, we refer to this term as *the follower's option value of firm i.*

- Equation (7b) corresponds to the case where firm i observes a signal greater than $\widehat{\sigma}_P$ and therefore does not want to participate to the first round of the pool whereas its opponent submits a bid smaller than $\overline{P}_0$. Firm i as the pool follower agrees to re-enter in case it is profitable. In such case, it serves the remaining capacity $\beta - \overline{\beta}$ at firm j's proposed price $P^P(s_j)$. We refer to this term as *the re-entry option value of firm i.*

Incentive compatibility requires that bidders with signals greater than $\widehat{\sigma}^P$ prefer not to bid to submitting the bid $\overline{P}_0$.

$$\left(1 - G\left(\widehat{\sigma}^P|\widehat{\sigma}^P\right)\right)\mathbb{E}\left[\overline{P}_0 - p(\widehat{\sigma}^P, S_j)|S_j > \widehat{\sigma}^P\right] = 0. \tag{8}$$

Lemma 1 *The threshold $\widehat{\sigma}^P$ exists and is unique. Moreover, $\widehat{\sigma}^P < \widetilde{\sigma}$.*

Given the specific rules of the pool (uniform pricing, options to exit and to re-enter), the follower profit is the same whatever the initial choice to participate to the auction. At $s_i = \widehat{\sigma}^P$, the follower's option value exactly compensates the re-entry option value: an insurer is indifferent between entering as a follower in the first round or re-entering in the second round. Therefore, the threshold is only determined by the condition that the leader's expected profit is non negative which refrains from bidding when signals are too high. We therefore say that insurers are *conservative.*

We look for an equilibrium strategy such that firm i's has an incentive to submit a bid according to its true signal . This implies that the equilibrium bid $P^P(s_i)$ satisfies the following differential equation[8]

$$P^{P'}(s_i) = \kappa \frac{g(s_i|s_i)}{1 - G(s_i|s_i)}\left(P^P(s_i) - p(s_i, s_i)\right), \forall s_i \leq \widehat{\sigma}^P. \tag{9}$$

[8]The details of the equilibrium analysis is in the Appendix.

The differential equation (9) is solved with the boundary condition that $P^P(\widehat{\sigma}^P) = \overline{P}_0$. In order the bidding strategy to be strictly increasing, a necessary condition is that $\widehat{\sigma}^P \leq \widetilde{\sigma}$ (it is satisfied as Lemma 1 teaches us). We then obtain the following equilibrium strategy.

Proposition 1 *There exists a unique symmetric Nash equilibrium in strictly increasing equilibrium bidding strategies where*

$$P^P(s) = \overline{P}_0(1 - L(\widehat{\sigma}^P|s)) + \int_s^{\widehat{\sigma}^P} p(x,x)dL(x|s) \ \ \forall \ \ s \leq \widehat{\sigma}^P \tag{10}$$

with

$$L(x|s) = 1 - \exp\left(-\kappa \int_s^x \frac{g(\tau|\tau)}{1 - G(\tau|\tau)} d\tau\right) \tag{11}$$

and where $L(x|s)$ is an increasing function with $L(s|s) = 0$ and $L(1|s) = 1$.

One of the specificities of the pool is not only that insurer i may want to enter in the second round when it observes a signal $s_i > \widehat{\sigma}^P$, but also that it may decide not to participate to the auction (ex post) if its payoff is negative when it receives a signal $s_j \leq s_i \leq \widehat{\sigma}^P$ (and turns out to be the follower). This happens when $P^P(s_j) < p(s_i, s_j)$ with $s_j < s_i$. In this case, the capacity is not fully served leading to a market failure.

Because p is increasing in each of its argument, the leader signal's values for which the follower is indifferent between entering in the second round or not entering define a function $\overline{s}_j^P(s_i)$ for $s_i \in [0, \widehat{\sigma}^P]$ that takes value on $[\widehat{\sigma}^P, 1]$. It is implicitly defined by

$$P^P(s_i) = p(s_i, \overline{s}_j^P(s_i)). \tag{12}$$

Re-entry for firm j occurs if and only if $s_j \in (\widehat{\sigma}^P, \overline{s}_j^P(s_i))$.

Let us describe Figure 1 focusing on the case where insurer i's signal s_i is the smallest and is smaller than $\widehat{\sigma}^P$:

- in region I^P, $s_i \leq s_j \leq \widehat{\sigma}^P$: both insurance companies bid in the first round and never withdraw, total capacity is insured;

- in region II^P, $s_i \leq \widehat{\sigma}^P$, $\widehat{\sigma}^P \leq s_j \leq \overline{s}_j^P(s_i)$ (so that $P^P(s_i) > p(s_i, s_j)$): insurer i bids in the first round and insurer j enters in the second round, total capacity is insured;

- in region III^P, $s_i \leq \widehat{\sigma}^P$, $s_j \geq \widehat{\sigma}^P$ and $s_j > \overline{s}_j^P(s_i)$ (so that $P^P(s_i) < p(s_i, s_j)$): insurer i bids in the first round and insurer j does not participate to the pool, only

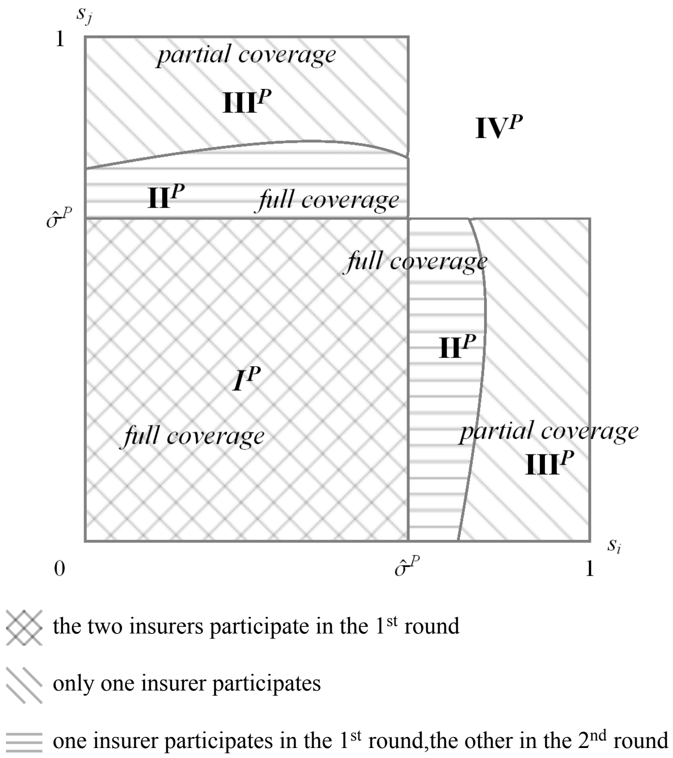


Figure 1: The different regions.

the leader provides capacity $\overline{\beta}$. We refer to this region as the *partial market failure region*;

- the boundary between regions II^P and III^P is $\left\{(s_i, s_j) \in [0,1]^2 | P^P(s_i) = p(s_i, s_j)\right\}$ and corresponds to $\overline{s}_j^P(s_i)$: insurer j is indifferent between entering in the second round and never participating to the pool;[9]

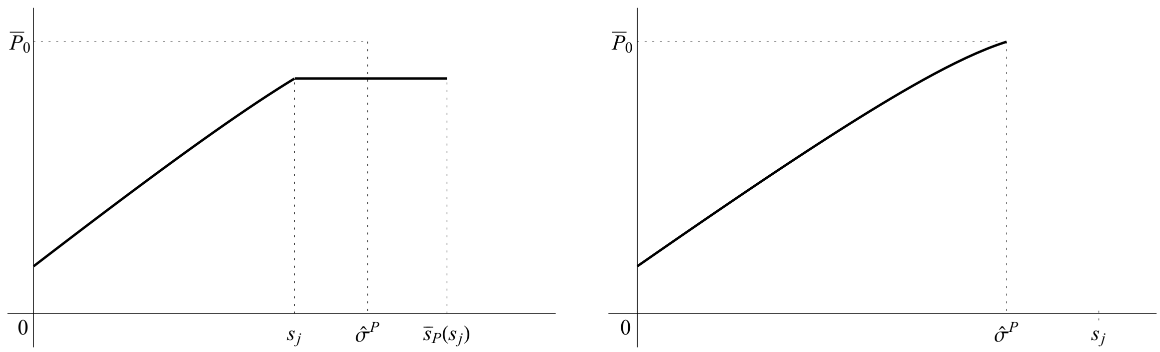
- in region IV^P, the two insurers observe a signal greater than $\widehat{\sigma}^P$, none of them submits a bid and no trade occurs. We refer to this region as the *complete market failure region*.

Note that the boundary between regions II^P and III^P, $\overline{s}_j^P(s_i)$, might be non-monotonic with respect to s_i. In particular, as $\frac{\partial^2 P^P(s_i)}{\partial s_i \partial \kappa} \geq 0$, the higher κ, the steeper $P^P(s_i)$. Therefore, if $s_i \mapsto P^P(s_i) - p(s_i, s_j)$ is a decreasing function of s_i when $\kappa = 0$; it might be a non monotonic function of s_i when κ is close to 1 as the Figure 1 illustrates.

P^P is the equilibrium bidding strategy. As for the premium, it depends on the signal

[9]Note that re-entry always exists: $P^P(\widehat{\sigma}^P) = \overline{P}_0 > p(\widehat{\sigma}^P, \widetilde{\sigma})$.

of the two insurers. The premium insurer i receives also depends on insurer j's signal as we can see in Figure 3.2.



(a) $s_j < \widehat{\sigma}^P$.

(b) $s_j > \widehat{\sigma}^P$.

Figure 2: Insurer i's premium for different insurer j's signal values.

On Figure 2(a), insurer i turns out to be the leader when $s_i \leq s_j$ in which case its premium corresponds to its bid, but when $s_i > s_j$, as the pool follower, its premium corresponds to insurer j's bid. Observe that insurer i decides to re-enter in the second round when its signal belongs to $[\widehat{\sigma}^P, \overline{s}^P(s_j)]$. On Figure 2(b), its opponent observes a signal high than $\widehat{\sigma}^P$, so that it is absent from the first round and insurer i is always the pool leader.

3.3 Equilibrium properties

Increasing competition. The region of the signal values for which insurers decide to submit a bid in the first round is independent of the strength of competition (see equation (8)). Being a follower in the first round or in the second round yields exactly the same (non-negative) profit. Then, $\widehat{\sigma}^P$ only matters for the leader's strategy and is determined to guarantee that the **unit** maximum net expected benefit is non negative. As a consequence, $\widehat{\sigma}^P$ does not depend on κ and so region I_i^P (where the total capacity is insured) and region IV_i^P (where there is complete market failure). However, the value of κ modifies the equilibrium bid P^P which in turn affects the follower decision to enter or not in the second round (the boundary between regions II_i^P and III_i^P).

Proposition 2 *When competition increases*

- *Region II_i^P (resp. III_i^P) shrinks (resp. expands);*

- *The equilibrium bidding strategy $P^P(s)$ decreases.*

The equilibrium bidding strategy is represented in Figure 3 for two values of κ. Competition unambiguously lowers premiums. When competition increases, the pool more often fails in offering complete coverage: the partial market failure region increases. However the proportion of the risk insured ($\frac{\bar{\beta}}{\beta} = \frac{1}{2-\kappa}$) increases. Therefore, increasing competition has two opposite effects on coverage: partial coverage is more likely but the proportion of uninsured risk decreases.

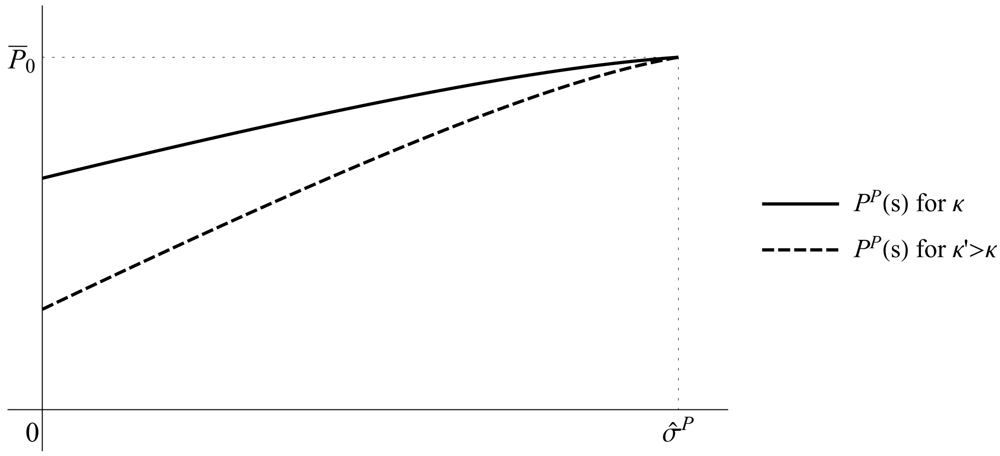


Figure 3: The equilibrium bidding strategy $P^P(s)$ for two values of κ.

Modifying the reserve price. A change in the reserve price can also be interpreted as an increase in insurees' risk aversion (see Gollier (????)).

Proposition 3 *When the reserve price increases*

- *Region I_i^P (resp. IV_i^P) expands (resp. shrinks)*

- *The equilibrium bidding strategy P^P increases.*

If a higher reserve price unambiguously increases the bidding regions, it has an ambiguous effect on the pool premium. Indeed, on the one hand, for a given bidding region (direct effect), a greater reserve price tends to increase the premium insurance companies may ask to the insuree. But on the other hand, as bidding regions increase and because the equilibrium is in strictly increasing strategies, this tends to lowers the premiums (indirect effect). However, we show that the direct effect always dominates so that the more risk averse insurees (the higher the reserve price, the higher the equilibrium bidding strategy as Figure 4 illustrates.

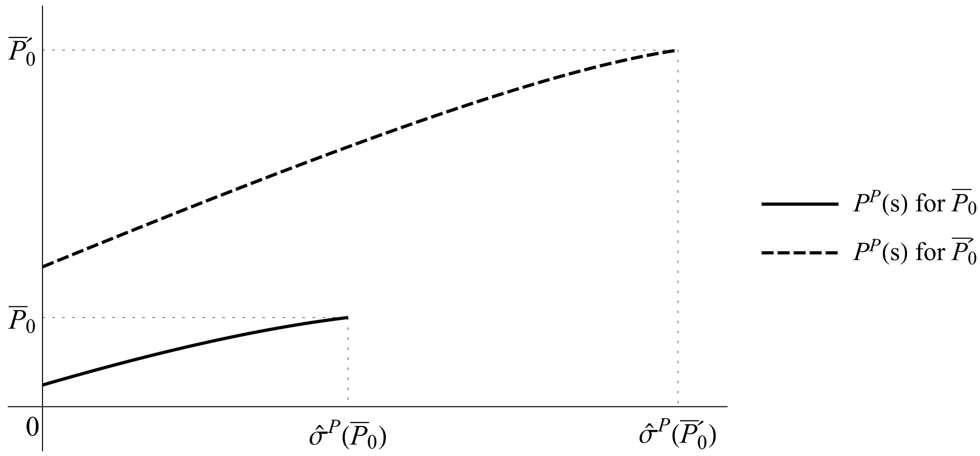


Figure 4: The equilibrium bidding strategy $P^P(s)$ for two values of $\overline{P}_0$.

Increasing the common knowledge of risk. We assume that the signal is a weighted average of a common signal and a private signal, each following a uniform distribution function on $[0,1]$

$$\begin{aligned} S_i &= (1-\gamma)Y_i + \gamma\varepsilon \\ S_j &= (1-\gamma)Y_j + \gamma\varepsilon \end{aligned}$$

where Y_i, Y_j and ε are three independent random variables distributed according to a uniform distribution on [0,1]. When $\gamma = 0$, the common term of the signals disappears so that the two signals are independent (this corresponds to the case where the risk to be insured is new). On the contrary, when $\gamma = 1$, the two signals equal ε (this corresponds to the case where the risk is already known). Therefore, by increasing γ , we describe a situation where insurers have more and more common information on the risk. Parameter γ can be interpreted as the degree of common knowledge: new risks may be associated with low value of γ.

The computation of the joint density function is given in Appendix.

Proposition 4 *When the degree of common knowledge γ increases*

- *Region I_i^P (resp. IV_i^P) expands (resp. shrinks);*

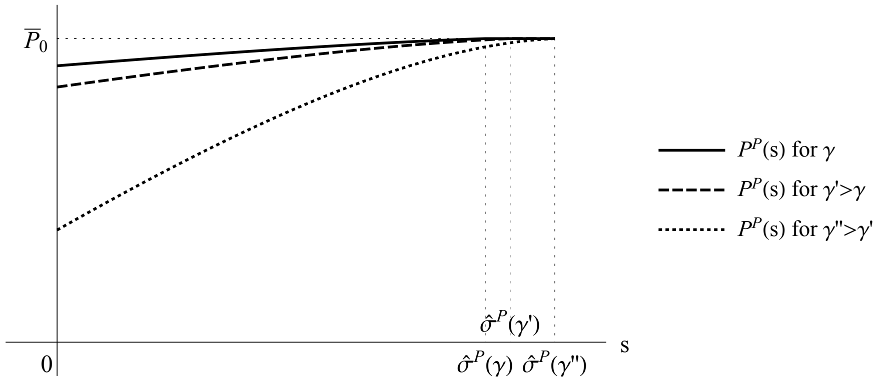


Figure 5: Leader's premium in the pool for three different values of γ.

- *The equilibrium bidding strategy $P^P(s)$ decreases.*

A change in γ has two opposite effects on the supply of insurance. On the one hand, there is a higher complete market failure (no insurance) for new risks ($\widehat{\sigma}^P$ decreases when γ decreases). On the other hand, a direct consequence of the decrease in the equilibrium strategies is that the boundary between regions II^P and III^P decreases with γ. This means that, conditional on at least one optimistic insurer ($s_i \leq \widehat{\sigma}^P$), full coverage increases for new risks (partial coverage decreases). Concerning known risks, the possibility to end up with no insurance decreases but we may observe more partial coverage. To sum-up, new risks (known risks) lead to a more (lower) complete market failure but the partial market failure reduces (increases).

4 A market alternative: A discriminatory auction

As an alternative to the pool, we model an agreement between insurance companies without premiums alignment. This corresponds to the case where a broker collects all the information and leaves the insurer with the lowest signal in the syndicate's leadership. The bidding process is organized in a single round where each firm proposes a risk premium according to the private signal it received. The leader and the follower (if any) sell insurance coverage at their announced risk premium. Typically, this corresponds to a discriminatory auction. We first provide a detailed equilibrium analysis and we turn to the comparison with the pool.

4.1 Equilibrium analysis

Separating equilibrium We first look for an equilibrium in strictly increasing and symmetric bidding strategies. Proceeding as for the pool, assume there exists a threshold $\widehat{\sigma}^D$ such that

- when $s_i \leq \widehat{\sigma}^D$, firm i bids according to a strictly increasing bidding strategy $P^D(s_i)$ with $P^D(\widehat{\sigma}^D) = \overline{P}_0$,
- when $s_i > \widehat{\sigma}^D$, firm i does not participate anymore.

The profit of firm i that observed a signal s_i and bids a risk premium $P^D(s_i)$ reads

$$\Pi^D(s_i) = \begin{cases} \overline{\beta}\left(1 - G\left(s_i|s_i\right)\right) \mathbb{E}\left[P^D(s_i) - p(s_i, S_j)|S_j > s_i\right] & \\ \quad + \left(\beta - \overline{\beta}\right) G\left(s_i|s_i\right) \mathbb{E}\left[P^D(s_i) - p(s_i, S_j)|S_j < s_i\right] & \text{for } s_i \leq \widehat{\sigma}^D \quad (13a) \\ 0 & \text{for } s_i > \widehat{\sigma}^D. \quad (13b) \end{cases}$$

As for the pool, the *leader's value* (first term of equation (13a)) correspond to the case where firm i proposes the smallest risk premium. Unlike the pool, the *follower's value* (second term of equation (13a)) now depends on the follower's premium and not on the leader's premium. It tends therefore to be greater than the pool's follower's value.

Incentive compatibility requires that bidders with signals greater than $\widehat{\sigma}^D$ prefer not to bid to submitting the bid $\overline{P}_0$.

$$\left(1 - G\left(\widehat{\sigma}^D|\widehat{\sigma}^D\right)\right) \mathbb{E}\left[\overline{P}_0 - p(\widehat{\sigma}^D, S_j)|S_j > \widehat{\sigma}^D\right] + (1-\kappa)\, G\left(\widehat{\sigma}^D|\widehat{\sigma}^D\right) \mathbb{E}\left[\overline{P}_0 - p(\widehat{\sigma}^D, S_j)|S_j < \widehat{\sigma}^D\right] = 0. \tag{14}$$

Contrary to equation (8) that defined the pool threshold, the follower's payoff (the second term of equation (14)) matters. As a consequence, the leader's expected payoff is negative at the threshold making the winner's curse more intense. Indeed, an insurer bids until $\widehat{\sigma}^D$ in the expectation of being the follower rather than the leader. As a consequence, the solution of Equation (14) may be larger than $\widetilde{\sigma}$.

At equilibrium, $P^D(b) = P^D(s_i), \forall s_i \leq \widehat{\sigma}^D$ so that

$$\frac{\partial \Pi_i^D(b, s_i)}{\partial b}|_{b=s_i} = 0.$$

As for the pool, incentive compatibility constraints imply that the equilibrium bid $P^D(s_i)$ satisfies the following differential equation solved with the boundary condition

$P^D(\widehat{\sigma}^D) = \overline{P}_0$.

$$P^{D\prime}(s_i) = \frac{\kappa g(s_i|s_i)}{1-\kappa G(s_i|s_i)} \left(P^D(s_i) - p(s_i, s_i)\right), \tag{15}$$

and we must have $\widehat{\sigma}^D \leq \widetilde{\sigma}$ in order the bidding strategy to be strictly increasing.

Lemma 2 *If* $\mathbb{E}\left[\overline{P}_0 - p(\widetilde{\sigma}, S_j)\right] - \kappa G\left(\widetilde{\sigma}|\widetilde{\sigma}\right) \mathbb{E}\left[\overline{P}_0 - p(\widetilde{\sigma}, S_j)|S_j < \widetilde{\sigma}\right] \leq 0$, *there exists a unique threshold* $\widehat{\sigma}^D$ *on* $[0, \widetilde{\sigma}]$.

Whether $\widehat{\sigma}^D \leq \widetilde{\sigma}$ now depends on the parameters of the model (the shape of the actuarial premium rate p, the conditional probability distribution characterized by the density probability distribution g and the cumulative probability distribution G, the strength of competition κ and the reserve price $\overline{P}_0$).[10]

Semi-pooling equilibrium. If $\mathbb{E}\left[\overline{P}_0 - p(\widetilde{\sigma}, S_j)\right] - \kappa G\left(\widetilde{\sigma}|\widetilde{\sigma}\right) \mathbb{E}\left[\overline{P}_0 - p(\widetilde{\sigma}, S_j)|S_j < \widetilde{\sigma}\right] > 0$, we must look for another equilibrium strategy that involves pooling for some values of the signal. More precisely, we look for an equilibrium in symmetric and increasing bidding strategy that is characterized by two thresholds $\underline{\sigma}^D$ and $\overline{\sigma}^D > \underline{\sigma}^D$ such that

- when $s_i \in [0, \underline{\sigma}^D]$, firm i bids according to a strictly increasing bidding strategy $P^D(s_i)$ with $P^D(\underline{\sigma}^D) = \overline{P}_0$,
- when $s_i \in [\underline{\sigma}^D, \overline{\sigma}^D]$, firm i bids $\overline{P}_0$,
- when $s_i > \overline{\sigma}^D$, firm i does not participate anymore.

The equilibrium is thus separating when $s_i \in [0, \underline{\sigma}^D]$ and it is pooling when $s_i \in [\underline{\sigma}^D, \overline{\sigma}^D]$. The profit of firm i that received a signal s_i and proposes a risk premium $P^D(s_i)$ reads

$$\Pi^D(s_i) = \begin{cases} \overline{\beta}\left(1 - G\left(s_i|s_i\right)\right) \mathbb{E}\left[P^D(s_i) - p(s_i, S_j)|S_j > s_i\right] & \\ \quad + \left(\beta - \overline{\beta}\right) G\left(s_i|s_i\right) \mathbb{E}\left[P^D(s_i) - p(s_i, S_j)|S_j < s_i\right] & \text{for } s_i \leq \underline{\sigma}^D \quad \text{(16a)} \\ \overline{\beta}\left(1 - G\left(\overline{\sigma}^D|\overline{\sigma}^D\right)\right) \mathbb{E}\left[P^D(s_i) - p(s_i, S_j)|S_j > \overline{\sigma}^D\right] & \\ \quad + \frac{\beta}{2}\left(G\left(\overline{\sigma}^D|\overline{\sigma}^D\right) - G\left(\underline{\sigma}^D|\underline{\sigma}^D\right)\right) \mathbb{E}\left[P^D(s_i) - p(s_i, S_j)|\underline{\sigma}^D < S_j < \overline{\sigma}^D\right] & \text{for } \underline{\sigma}^D < s_i \leq \overline{\sigma}^D \\ \quad + \left(\beta - \overline{\beta}\right) G\left(\underline{\sigma}^D|\underline{\sigma}^D\right) \mathbb{E}\left[P^D(s_i) - p(s_i, S_j)|S_j < \underline{\sigma}^D\right] & \text{(16b)} \\ 0 & \text{for } s_i > \overline{\sigma}^D. \quad \text{(16c)} \end{cases}$$

[10]See Proposition 6 for a comparative static analysis of $\widehat{\sigma}^D$ with respect to κ.

Contrary to (13), there is a new intermediate case where firm i bids $\overline{P}_0$ (equation (16b)). The first term corresponds to the *leader's value*, the last to the *follower value.* As for the second term, it corresponds to the case where the two firms bid $\overline{P}_0$ so that they equally share the market.

Incentive compatibility requires that insurers with signal in $[\underline{\sigma}^D, \overline{\sigma}^D]$ prefer submitting $\overline{P}_0$ to not participating and to submitting any lower bid. Moreover, insurers with signals greater than $\overline{\sigma}^D$ prefer not to bid to submitting the bid $\overline{P}_0$. The two thresholds are thus defined by the following system.

$$\begin{cases} \left(G\left(\overline{\sigma}^D|\overline{\sigma}^D\right) - G\left(\underline{\sigma}^D|\underline{\sigma}^D\right)\right)\mathbb{E}\left[\overline{P}_0 - p(s_i, S_j)|\underline{\sigma}^D < S_j < \overline{\sigma}^D\right] = 0 & \text{(17a)} \\ \left(1 - G\left(\overline{\sigma}^D|\overline{\sigma}^D\right)\right)\mathbb{E}\left[\overline{P}_0 - p(s_i, S_j)|S_j > \overline{\sigma}^D\right] & \\ +\left(1 - \frac{\kappa}{2}\right)\left(G\left(\overline{\sigma}^D|\overline{\sigma}^D\right) - G\left(\underline{\sigma}^D|\underline{\sigma}^D\right)\right)\mathbb{E}\left[\overline{P}_0 - p(s_i, S_j)|\underline{\sigma}^D < S_j < \overline{\sigma}^D\right] & \\ +G\left(\underline{\sigma}^D|\underline{\sigma}^D\right)\mathbb{E}\left[\overline{P}_0 - p(s_i, S_j)|S_j < \underline{\sigma}^D\right] = 0. & \text{(17b)} \end{cases}$$

It must also be checked that an insurer that bids $\overline{P}_0$ when it observes a signal comprised between $\underline{\sigma}^D$ and $\overline{\sigma}^D$ does not have an incentive to underprice. This comes down to checking that[11]

$$\left(G\left(\overline{\sigma}^D|\overline{\sigma}^D\right) - G\left(\underline{\sigma}^D|\underline{\sigma}^D\right)\right)\mathbb{E}\left[\overline{P}_0 - p(s_i, S_j)|\underline{\sigma}^D < S_j < \overline{\sigma}^D\right] \leq 0 \;\; \forall s_i \in [\underline{\sigma}^D, \overline{\sigma}^D].$$

Lemma 3 *The semi-pooling equilibrium exists and is unique if and only if the separating equilibrium does not exist ($\widehat{\sigma}^D > \widetilde{\sigma}$). Moreover, if the semi-pooling equilibrium exists, the following ranking holds*

$$\alpha(\overline{\sigma}^D) \leq \underline{\sigma}^D < \widetilde{\sigma} < \alpha(\underline{\sigma}^D) \leq \overline{\sigma}^D \leq \widehat{\sigma}^D.$$

When the separating equilibrium exists ($\widehat{\sigma}^D < \widetilde{\sigma}$), the system ((17a)-(17b)) has a unique solution $\underline{\sigma}^D = \overline{\sigma}^D = \widehat{\sigma}^D$ involving no pooling region. We can then state the following proposition that characterizes the equilibrium strategy.

Proposition 5 *If $\widehat{\sigma}^D \leq \widetilde{\sigma}$, there exists a unique separating symmetric Nash equilibrium.*

[11]This is checked in the proof of Lemma 3.

The strictly increasing bidding equilibrium strategies read

$$P^D(s) = \overline{P}_0(1 - K(\widehat{\sigma}^D|s)) + \int_s^{\widehat{\sigma}^D} p(x,x)dK(x|s) \ \ \forall s \leq \widehat{\sigma}^D. \tag{18}$$

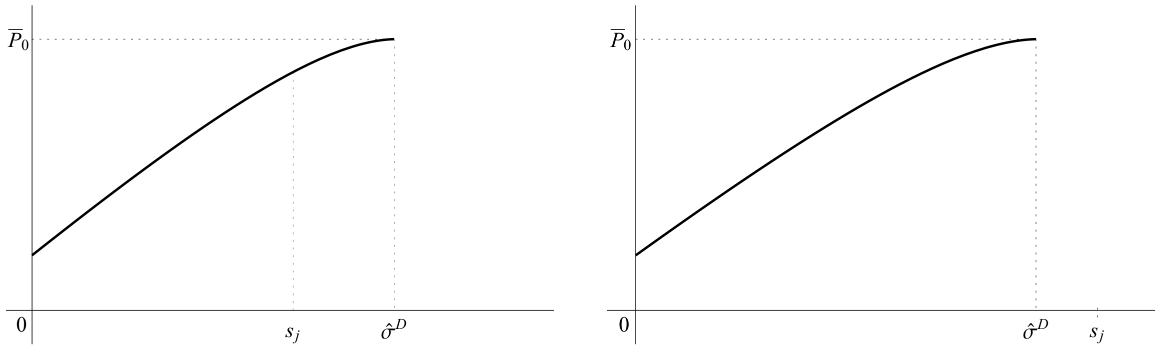
If $\widehat{\sigma}^D > \widetilde{\sigma}$, there exists a unique symmetric Nash equilibrium in increasing bidding equilibrium strategies with partial pooling where

$$P^D(s) = \begin{cases} \overline{P}_0(1 - K(\underline{\sigma}^D|s)) + \int_s^{\underline{\sigma}^D} p(x,x)dK(x|s) & \text{for } s \leq \underline{\sigma}^D \quad \text{(19a)} \\ \overline{P}_0 & \text{for } \underline{\sigma}^D < s \leq \overline{\sigma}^D \quad \text{(19b)} \end{cases}$$

where

$$K(x|s) = 1 - \exp\left(-\int_s^x \frac{\kappa g(\tau|\tau)}{1 - \kappa G(\tau|\tau)} d\tau\right) \tag{20}$$

is an increasing function with $K(s|s) = 0$ and $K(1|s) \leq 1$.



(a) $s_j < \widehat{\sigma}^P$. (b) $s_j > \widehat{\sigma}^P$.

Figure 6: Insurer i's premium for different insurer j's signal values.

4.2 Equilibrium properties

This section provides a comparative static analysis of the discriminatory auction equilibrium where we emphasize the role of the strength of competition and the reserve price on the equilibrium outcome.

Contrary to the pool, the region in which insurance companies submit bids now depends on the competition strength. The thresholds matter not only for the leader's but also for the follower's strategy so that the leader and the follower capacities (and thus the strength of competition) are important to determine the bidding regions strategies. The higher κ, the lower the capacity is left to the follower. Therefore, the three thresholds $\widehat{\sigma}^D$, $\underline{\sigma}^D$ and $\overline{\sigma}^D$ depend on κ.

Lemma 4 *In the separating equilibrium, the signal limiting the bidding region is decreasing in κ and increasing in $\overline{P}_0$*

$$\frac{\partial \widehat{\sigma}^D}{\partial \kappa} \leq 0 \text{ and } \frac{\partial \widehat{\sigma}^D}{\partial \overline{P}_0} \geq 0.$$

In the semi-pooling equilibrium, the lowest (resp. highest) bound of the pooling region is increasing (resp. decreasing) in κ

$$\frac{\partial \underline{\sigma}^D}{\partial \kappa} \geq 0 \text{ and } \frac{\partial \overline{\sigma}^D}{\partial \kappa} \leq 0.$$

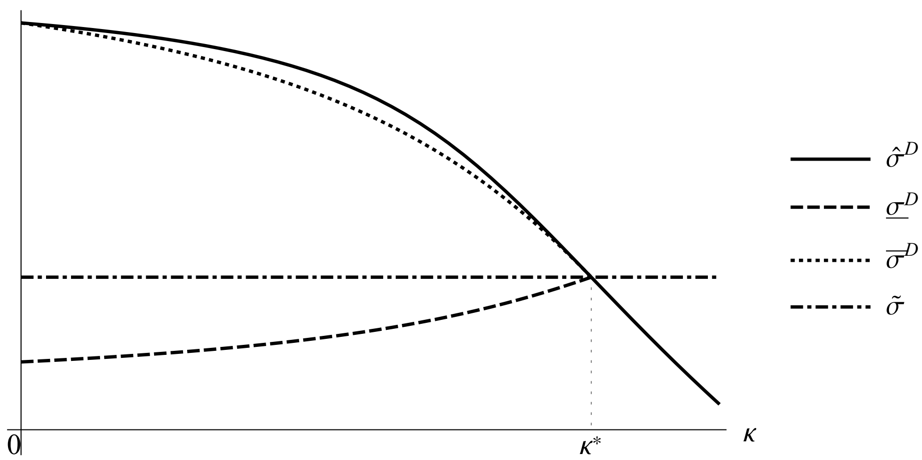


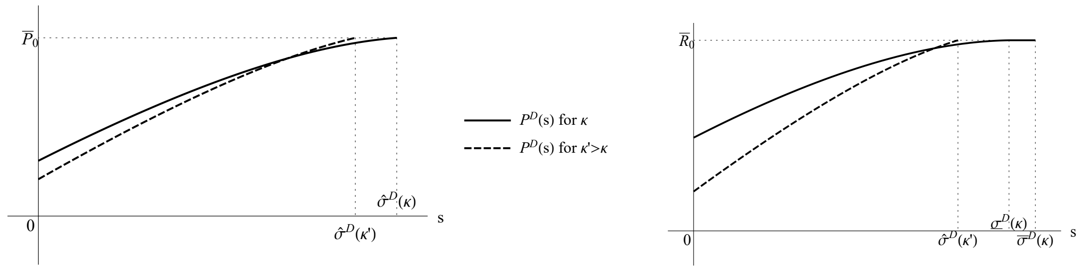
Figure 7: The different thresholds as a function of κ.

The regions in which the capacity is fully served are decreasing with competition. The minimal signal for which insurers bid the reserve price is non monotonic with respect to κ.

We now discuss how equilibrium bidding strategies are affected by a change in the parameters' values. Firstly, as for the pool, the equilibrium bidding strategy P^D is increasing in $\overline{P}_0$. Secondly, when there is full coverage, the comparative statics with respect to κ is not straightforward for the premium P^D. Indeed, even if the direct effect that tends to decrease the premium level is the same as for the pool, there exists an indirect effect that comes from the fact that the threshold $\min(\widehat{\sigma}^D, \underline{\sigma}^D)$ depends on κ. This indirect effect writes:

$$-\frac{\partial \min(\widehat{\sigma}^D, \underline{\sigma}^D)}{\partial \kappa} \left(\overline{P}_0 - p(\min(\widehat{\sigma}^D, \underline{\sigma}^D), \min(\widehat{\sigma}^D, \underline{\sigma}^D))\right) \frac{dK(x|s)}{dx}|_{x=\min(\widehat{\sigma}^D, \underline{\sigma}^D)}.$$

If $\min(\widehat{\sigma}^D, \underline{\sigma}^D) = \underline{\sigma}^D$ (when the semi-pooling equilibrium exists, $\kappa < \kappa^*$), the indirect effect is negative as the direct effect. In this case, P^D decreases when competition increases until κ^*. At the opposite, the indirect effect is positive in the separating equilibrium since $\widehat{\sigma}^D$ is a decreasing function of κ. As we can see in Figure 8(a), the total effect is ambiguous. Also, the variation in premium is ambiguous when κ increases so that the equilibrium switches from semi-pooling to separating as we can see in Figure 8(b).



(a) $\kappa^* < \kappa < \kappa'$.

(b) $\kappa < \kappa^* < \kappa'$.

Figure 8: Premiums in the two equilibria.

A direct consequence of these discussions is that we can identify whether the equilibrium is separating or semi-pooling depending on both κ and $\overline{P}_0$.

Proposition 6 *There exists an increasing function*

$$\kappa^*\left(\overline{P}_0\right) = \max\left(\frac{\overline{P}_0 - \mathbb{E}\left[p\left(\widetilde{\sigma}, S_j\right)\right]}{G\left(\widetilde{\sigma}|\widetilde{\sigma}\right)\left(\overline{P}_0 - \mathbb{E}\left[p\left(\widetilde{\sigma}, S_j\right)|S_j < \widetilde{\sigma}\right]\right)}, 0\right) \tag{21}$$

such that the separating equilibrium (resp. the semi-pooling equilibrium) exists if and only if $\kappa \geq \kappa^*(\overline{P}_0)$ *(resp.* $\kappa < \kappa^*(\overline{P}_0)$*).*

When $\overline{P}_0 \leq \mathbb{E}\left[p(S_i, \widetilde{\sigma})\right]$ the reserve price is too low with respect to the expected actuarial premium, so that bidders refrain from insuring large risks and $\widehat{\sigma}^D < \widetilde{\sigma}$. Then, $\kappa^*\left(\overline{P}_0\right) = 0$ and the separating equilibrium always exists. On the contrary, when $\overline{P}_0 > \mathbb{E}\left[p(S_i, \widetilde{\sigma})\right]$, we have that $\kappa^*\left(\overline{P}_0\right) > 0$ (as illustrated in Figure 7). The semi-pooling equilibrium exists when $\kappa \leq \kappa^*(\overline{P}_0)$ and the separating equilibrium exists when $\kappa \geq \kappa^*(\overline{P}_0)$. In the semi pooling equilibrium, the outcome of the pooling region $[\underline{\sigma}^D, \overline{\sigma}^D]$ is the monopoly outcome $(\overline{P}_0)$. The lower the competition, the larger this region. When competition becomes too intense, this region disappears and the separating equilibrium exists. In this case, the capacity left to the follower decreases so that bidders refrain from taking high risks ($\widehat{\sigma}^D$ decreases).

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5 Which auction for the coverage of these risks?

We evaluate the efficiency of the pool with respect to two dimensions: premiums' levels and coverage. Strength of competition, insuree's risk aversion and affiliation are the key elements in this discussion.

5.1 Pool vs discriminatory auction

We have already explained that both auctions deal with different risk/return tradeoff for the follower. The follower's position is essential to understand the efficiency of an auction format. In the pool, the follower does not take any risk, but only enjoys relatively low profits (because of uniform pricing), whereas in the discriminatory auction the potential negative profit of the follower is counterbalanced by higher premiums.

This first result compares the different thresholds.

Lemma 5 *Insurance companies submit the maximal bid $\overline{P}_0$ for lower signal's values in the pool auction*

$$\widehat{\sigma}^P \leq \min\left(\widehat{\sigma}^D, \underline{\sigma}^D\right).$$

Lemma 5 means that regions I^P are always smaller than regions I^D. If both signals are between $\widehat{\sigma}^P$ and $\widehat{\sigma}^D$, the discriminatory auction offers a full coverage of the risk whereas the pool offers no coverage. If one signal is between $\widehat{\sigma}^P$ and $\widehat{\sigma}^D$ and the other is larger than $\widehat{\sigma}^D$, the pool offers no coverage and the discriminatory auction offer a partial coverage. When only one insurer initially bids ($s_j > \widehat{\sigma}^D > s_i$), the pool may offer full coverage (in region II_i^P) whereas the other auctions always cover risk only partially. Even if insurers are more conservative in the pool $\left(\widehat{\sigma}^P \leq \min\left(\widehat{\sigma}^D, \underline{\sigma}^D\right)\right)$, the re-entry option left to the follower eliminates its ex post risk so that it can re-enter after having observed a signal larger than $\widehat{\sigma}^D$.

When competition increases, the difference in coverage between the two auctions reduces.[12]

[12] Both $\widehat{\sigma}^D$ and $\overline{\sigma}^D$ decrease with κ so that the discriminatory auction additional full coverage area decreases. $\widehat{\sigma}^P$ is independent of κ, but the frontier determining the follower re-entry region decreases, diminishing the additional coverage providing by the pool.

The comparison of the equilibrium bidding strategies is stated in the following lemma.

Lemma 6 *If $P^D(0) \le P^P(0)$, then $P^D(s) < P^P(s)$ for all $s \in (0, \widehat{\sigma}^P]$.*
If $P^D(0) > P^P(0)$, there exists a unique $\breve{s} \in (0, \widehat{\sigma}^P]$, such that $P^P(s) < P^D(s)$ if and only if $s < \breve{s}$.

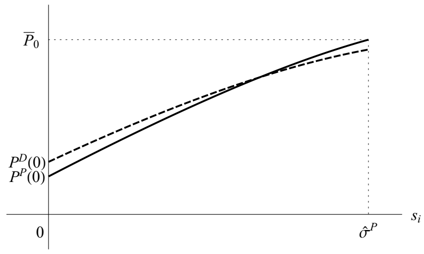
This result is driven both by the conservativeness of the bidding interval in the pool and by the marginal effect of an increase in risk on bidding strategies. First, as $\overline{P}_0$ is reached for smaller signal values in the pool, equilibrium bidding strategies are higher in the pool than in the discriminatory auction for signal values close to $\widehat{\sigma}^P$. Indeed, an insurance company receiving a high signal knows that it faces no risk by being the pool follower whereas it faces the risk of receiving a negative revenue as the pool leader. Thus, for such signal values that still belong to the bidding regions, players submit higher bids than in the discriminatory auction. Second, we look for increasing bidding strategies. Therefore, starting from the upper boundary, both P^D and P^P decrease when the signal decreases. Imagine there exists a signal value such that the bidding strategies are equal in the two auction formats. Having a look at the two ODEs satisfied by the strategies, we observe that the slope must be higher in the pool. Indeed, the expected revenue in the pool for a given insurer is only affected in the case it is the leader, whereas the expected revenue is affected whatever the insurer's role in the discriminatory auction. Therefore, in order to compensate this asymmetry between the leader's and the follower's positions, the marginal bid increase is higher in the pool. However, the reserve price can be so low that such a threshold does not exist (when $P^P(0) > P^D(0)$).

Once again, the analysis must also be done for the premiums as we can see in Figure 9.

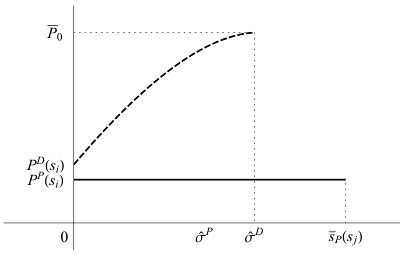
Clearly, uniform pricing at the leader's premium tends to reduce the expected value of the premium. Nevertheless, when both signals are high (so that $P^D(s_i) < P^P(si)$ when insurer i is the leader), the ranking of the follower's premiums can be reversed. Indeed, in the discriminatory auction, conditional on bidding despite a high signal, players tend to submit high bids to ensure a high potential follower's revenue.

Comparison with respect to affiliation.

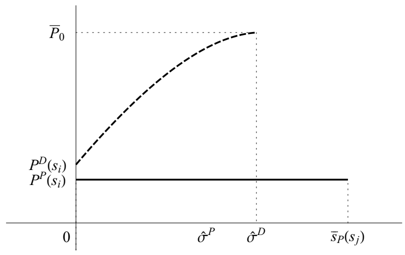
This analysis shows that ex-ante there is no clear dominance of one auction format. However, we can highlight two messages depending on the realization of the signals. If at



(a) Leader's premium.



(b) Follower's premium.



(c) Follower's premium ERREUR.

Figure 9: Comparisons of the premiums.

least one insurer provides an optimistic expertise, the pool offers both lower premiums and higher coverage. This is true whatever the ex-post expertise of the other insurer. When all insurers agree about an ex-post pessimistic evaluation of the risk, the discriminatory auction offers a better coverage at a lower leader's premium, but a higher follower's premium.

5.2 What about re-entry?

Two major differences exist between the two settings. The follower pricing rule (uniform versus heterogenous pricing) and the re-entry option. In order to better understand the effect of each characteristics, we now introduce a re-entry option in the discriminatory auction, that is the possibility for an insurer that did not participate in the first round (because it was too pessimistic) to re-enter and to sell at the leader's price.

We obtain an equilibrium[13] similar to the discriminatory auction equilibrium except for the different thresholds. In particular, the threshold $\widehat{\sigma}_r^D$ in the separating equilibrium is given by:

$$\begin{aligned} & \left(1 - G\left(\widehat{\sigma}_r^D | \widehat{\sigma}_r^D\right)\right) \mathbb{E}\left[\overline{P}_0 - p(\widehat{\sigma}_r^D, S_j) | S_j > \widehat{\sigma}_r^D\right] + (1-\kappa)\, G\left(\widehat{\sigma}_r^D | \widehat{\sigma}_r^D\right) \mathbb{E}\left[\overline{P}_0 - p(\widehat{\sigma}_r^D, S_j) | S_j < \widehat{\sigma}_r^D\right] \\ = \; & (1-\kappa)\, G\left(\widehat{\sigma}_r^D | \widehat{\sigma}_r^D\right) \mathbb{E}\left[\left(P_r^D(S_j) - p(\widehat{\sigma}_r^D, S_j)\right)_+ | S_j < \widehat{\sigma}_r^D\right]. \end{aligned} \tag{22}$$

[13]The equilibrium analysis is relegated to the Appendix.

The main difference with the conditions determining $\widehat{\sigma}^D$ (equation (14)) and $\widehat{\sigma}^P$ (equation (8) is that the equilibrium bidding strategy now enters into the determination of $\widehat{\sigma}_r^D$.[14]

Lemma 7 *Re-entry in the discriminatory auction yields to more conservative strategies ($\widehat{\sigma}^D \geq \widehat{\sigma}_r^D$).*
Heterogenous pricing yields to less conservative strategies when re-entry is allowed ($\widehat{\sigma}_r^D \geq \widehat{\sigma}^P$).

Not surprisingly, introducing the re-entry option in the discriminatory auction leads to more conservative strategies. Indeed, when re-entry is possible, an insurer takes less risks in its bidding strategy since submitting a bid is not a necessary condition to participate to the auction anymore. Therefore, an insurer reduces the risk of being a leader despite a high signal by reducing the bidding region.

When re-entry is possible, heterogenous pricing increases the bidding region. Being a follower after having bid in the first round leads to higher premiums than re-entering in the second round and being a follower at the leader's premium. Therefore, the re-entry option is less valuable under heterogenous pricing. As a consequence, insurers take advantage of the first round by enlarging the bidding region.

A re-entry option in an auction then yields to more complete market failure but reduces the partial market failure.

The comparison of the bidding strategies is the next natural step.

Lemma 8 *Re-entry in the discriminatory auction raises equilibrium bidding strategies: $P_r^D(s) > P^D(s)$, for all $s < \widehat{\sigma}_r^D$.*
The effect of uniform versus heterogenous pricing on equilibrium bidding strategies is ambiguous. If $P_r^D(0) \leq P^P(0)$, then $P_r^D(s) < P_r^P(s)$ for all $s \in (0, \widehat{\sigma}^P]$.
If $P_r^D(0) > P^P(0)$, there exists a unique $\breve{s}_r \in (0, \widehat{\sigma}^P]$, such that $P^P(s) < P_r^D(s)$ if and only if $s < \breve{s}_r$.

In the discriminatory auction, re-entry raises premium since re-entering followers benefit from the leader's revenue. When re-rentry is possible, the comparison between heterogenous and uniform pricing relies on the same forces than the ones described in Lemma ???. Also it is shown in appendix ???? that re-entry increases premium in the uniform

[14] The follower's value does not cancel out with the re-entry option value as in the pool.

auction. A re-entry option then yields to higher premiums both with heterogenous and uniform pricing.

6 Concluding remarks

A Determination of the equilibria

A.1 Equilibrium analysis of the pool

A.2 Equilibrium analysis of the discriminatory auction

A.3 Equilibrium analysis of the discriminatory auction with re-entry

A.3.1 Separating equilibrium.

Assume first there exists a threshold $\widehat{\sigma}_r^D$ such that

- when $s_i \leq \widehat{\sigma}_r^D$, firm i bids according to a strictly increasing bidding strategy $P_r^D(s_i)$ with $P_r^D(\widehat{\sigma}_r^D) = \overline{P}_0$,
- when $s_i > \widehat{\sigma}_r^D$, firm i may enter in the second round at the risk premium its opponent bids in case it is profitable.

The profit of firm i that observed a signal s_i and bids a risk premium $P_r^D(s_i)$ reads

$$\Pi_r^D(s_i) = \begin{cases} \overline{\beta}\,(1 - G(s_i|s_i))\,\mathbb{E}\left[P_r^D(s_i) - p(s_i, S_j)|S_j > s_i\right] \\ \quad + \left(\beta - \overline{\beta}\right) G(s_i|s_i)\,\mathbb{E}\left[P_r^D(s_i) - p(s_i, S_j)|S_j < s_i\right] & \text{for } s_i \leq \widehat{\sigma}_r^D \quad (23a) \\ \left(\beta - \overline{\beta}\right) G\left(\widehat{\sigma}_r^D|\widehat{\sigma}_r^D\right) \mathbb{E}\left[\left(P_r^D(S_j) - p(s_i, S_j)\right)_+ |S_j < \widehat{\sigma}_r^D\right] & \text{for } s_i > \widehat{\sigma}_r^D \quad (23b) \end{cases}$$

The threshold $\widehat{\sigma}_r^D$ is determined by the incentive compatibility constraint.[15]

$$\begin{aligned} & \left(1 - G\left(\widehat{\sigma}_r^D|\widehat{\sigma}_r^D\right)\right) \mathbb{E}\left[\overline{P}_0 - p(\widehat{\sigma}_r^D, S_j)|S_j > \widehat{\sigma}_r^D\right] + (1-\kappa)\, G\left(\widehat{\sigma}_r^D|\widehat{\sigma}_r^D\right) \mathbb{E}\left[\overline{P}_0 - p(\widehat{\sigma}_r^D, S_j)|S_j < \widehat{\sigma}_r^D\right] \\ = \; & (1-\kappa)\, G\left(\widehat{\sigma}_r^D|\widehat{\sigma}_r^D\right) \mathbb{E}\left[\left(P_r^D(S_j) - p(\widehat{\sigma}_r^D, S_j)\right)_+ |S_j < \widehat{\sigma}_r^D\right]. \end{aligned} \quad (24)$$

The only difference with Equation (14) comes from the re-entry option value term on the right hand side. Note that here, unlike the pool, the follower's value does not cancel out with the re-entry option value.

Lemma 9 *Re-entry at the leader's premium yields to more conservative strategies ($\widehat{\sigma}^D \geq \widehat{\sigma}_r^D \geq \widehat{\sigma}^P$).*

[15] This threshold is the maximal signal for which an insurer bids in the first round.

From a technical viewpoint, observe that Equation (24) determining $\widehat{\sigma}_r^D$ is much more complicated than Equation (14) determining $\widehat{\sigma}^D$ since the risk premium $P_r^D(s_j)$ that is determined at equilibrium and that depends on $\widehat{\sigma}_r^D$ enters in the expression. The analytical analysis is therefore much trickier.

A.3.2 Semi-pooling equilibrium.

If $\widehat{\sigma}_r^D > \widetilde{\sigma}$, the equilibrium strategy we just derived is not an equilibrium since it is not strictly increasing. We must look for another equilibrium strategy that involves pooling for some values of the signal. More precisely, we look for an equilibrium in symmetric and increasing bidding strategy that is characterized by two thresholds $\underline{\sigma}_r^D$ and $\overline{\sigma}_r^D > \underline{\sigma}_r^D$ such that

- when $s_i \in [0, \underline{\sigma}_r^D]$, firm i bids according to a strictly increasing bidding strategy $P_r^D(s_i)$ with $P_r^D(\underline{\sigma}_r^D) = \overline{P}_0$,
- when $s_i \in [\underline{\sigma}_r^D, \overline{\sigma}_r^D]$, firm i bids $\overline{P}_0$,
- when $s_i > \overline{\sigma}_r^D$, firm i does not participate in the first round anymore and may optionally re-enter in the second round.

The equilibrium is thus separating when $s_i \in [0, \underline{\sigma}_r^D]$ and it is pooling when $s_i \in [\underline{\sigma}_r^D, \overline{\sigma}_r^D]$. The profit of firm i that received a signal s_i and proposes a risk premium $P_r^D(s_i)$ reads

$$\Pi_r^D(s_i) = \begin{cases} \overline{\beta}\left(1 - G(s_i|s_i)\right)\mathbb{E}\left[P_r^D(s_i) - p(s_i, S_j)|S_j > s_i\right] \\ \quad + \left(\beta - \overline{\beta}\right) G(s_i|s_i)\,\mathbb{E}\left[P_r^D(s_i) - p(s_i, S_j)|S_j < s_i\right] & \text{for } s_i \leq \underline{\sigma}_r^D \quad \text{(25a)} \\ \overline{\beta}\left(1 - G\left(\overline{\sigma}_r^D|\overline{\sigma}_r^D\right)\right)\mathbb{E}\left[P_r^D(s_i) - p(s_i, S_j)|S_j > \overline{\sigma}_r^D\right] \\ \quad + \frac{\beta}{2}\left(G\left(\overline{\sigma}_r^D|\overline{\sigma}_r^D\right) - G\left(\underline{\sigma}_r^D|\underline{\sigma}_r^D\right)\right)\mathbb{E}\left[P_r^D(s_i) - p(s_i, S_j)|\underline{\sigma}_r^D < S_j < \overline{\sigma}_r^D\right] & \text{for } \underline{\sigma}_r^D < s_i \leq \overline{\sigma}_r^D \\ \quad + \left(\beta - \overline{\beta}\right) G\left(\underline{\sigma}_r^D|\underline{\sigma}_r^D\right)\mathbb{E}\left[P_r^D(s_i) - p(s_i, S_j)|S_j < \underline{\sigma}_r^D\right] & \text{(25b)} \\ \left(\beta - \overline{\beta}\right) G\left(\overline{\sigma}_r^D|\overline{\sigma}_r^D\right)\mathbb{E}\left[\left(P_r^D(S_j) - p(s_i, S_j)\right)_+ |S_j < \overline{\sigma}_r^D\right] & \text{for } s_i > \overline{\sigma}_r^D \quad \text{(25c)} \end{cases}$$

Contrary to (23), there is a new intermediate case where firm i bids $\overline{P}_0$ (equation (25b)). The first term corresponds to the leader's value, the last to the follower value. As for the second term, it corresponds to the case where the two firms bid $\overline{P}_0$ so that they equally share the market.

Incentive compatibility requires that insurers with signal in $[\underline{\sigma}_r^D, \overline{\sigma}_r^D]$ prefer submitting $\overline{P}_0$ to not participating and to submitting any lower bid. Moreover, insurers with signals greater than $\overline{\sigma}_r^D$ prefer not to bid to submitting the bid $\overline{P}_0$. The two thresholds are thus defined by the following system.

$$\begin{cases} \left(G\left(\overline{\sigma}_r^D|\overline{\sigma}_r^D\right) - G\left(\underline{\sigma}_r^D|\underline{\sigma}_r^D\right)\right)\mathbb{E}\left[\overline{P}_0 - p(s_i,S_j)|\underline{\sigma}_r^D < S_j < \overline{\sigma}_r^D\right] = 0 & \text{(26a)} \\ \left(1 - G\left(\overline{\sigma}_r^D|\overline{\sigma}_r^D\right)\right)\mathbb{E}\left[\overline{P}_0 - p(s_i,S_j)|S_j > \overline{\sigma}_r^D\right] & \\ +\left(1-\frac{\kappa}{2}\right)\left(G\left(\overline{\sigma}_r^D|\overline{\sigma}_r^D\right) - G\left(\underline{\sigma}_r^D|\underline{\sigma}_r^D\right)\right)\mathbb{E}\left[\overline{P}_0 - p(s_i,S_j)|\underline{\sigma}_r^D < S_j < \overline{\sigma}_r^D\right] & \text{(26b)} \\ +G\left(\underline{\sigma}_r^D|\underline{\sigma}_r^D\right)\mathbb{E}\left[\overline{P}_0 - p(s_i,S_j)|S_j < \underline{\sigma}_r^D\right] = (1-\kappa)\,G\left(\overline{\sigma}_r^D|\overline{\sigma}_r^D\right)\mathbb{E}\left[\left(P_r^D(s_j) - p(s_i,S_j)\right)_+ |S_j < \overline{\sigma}_r^D\right]. & \end{cases}$$

Proposition 7 *If $\widehat{\sigma}_r^D \leq \widetilde{\sigma}$, there exists a separating symmetric Nash equilibrium. The strictly increasing bidding equilibrium strategies read*

$$P_r^D(s) = \overline{P}_0(1 - K(\widehat{\sigma}_r^D|s)) + \int_s^{\widehat{\sigma}_r^D} p(x,x)dK(x|s) \ \ \forall s \leq \widehat{\sigma}_r^D. \tag{27}$$

If $\widehat{\sigma}_r^D > \widetilde{\sigma}$, there exists a symmetric Nash equilibrium in increasing bidding equilibrium strategies with partial pooling where

$$P_r^D(s) = \begin{cases} \overline{P}_0(1 - K(\underline{\sigma}_r^D|s)) + \int_s^{\underline{\sigma}_r^D} p(x,x)dK(x|s) & \text{for } s \leq \underline{\sigma}_r^D & \text{(28a)} \\ \overline{P}_0 & \text{for } \underline{\sigma}_r^D < s \leq \overline{\sigma}_r^D & \text{(28b)} \end{cases}$$

where $K(x|s)$ is defined by Equation (20).

Whether $\widehat{\sigma}_r^D$ is greater or smaller than $\widetilde{\sigma}$ depends on the parameters of the model, among them the strength of competition κ. However, the comparative statics with respect to κ, and thus whether the equilibrium is separating or semi-pooling, is more involved since the endogenous premium P_r^D enters in the definition of the thresholds. Note that in the two extreme cases where $\kappa = 0$ and $\kappa = 1$ Equation (24) implies that $\widehat{\sigma}_r^D = \widehat{\sigma}^P$ so that the separating equilibrium exists. Therefore, the separating equilibrium exists for low and large values of κ. The value of the re-entry option decreases with κ since both the premium and the residual capacity decrease. It is therefore less valuable to be the follower and insurers become less conservative. However, when competition becomes more intense, the winner's curse leads insurers to become more conservative. As a consequence, there is a non monotony in the shape of $\widehat{\sigma}_r^D$ as Figure 10 shows.

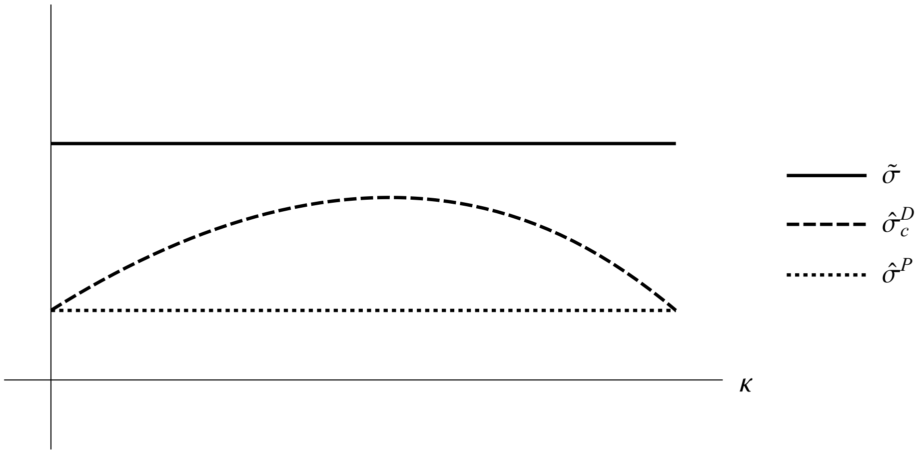


Figure 10: The different thresholds as a function of κ.

As we did for the pool let us define the maximal signal $\overline{s}_j^{D_r}$ for which the follower agrees to enter in the second round

$$P_r^D(s_i) = p(s_i, \overline{s}_j^{D_r}(s_i)). \tag{29}$$

It is defined for $s_i \in [0, \widehat{\sigma}_r^D]$ and takes values in $[\widehat{\sigma}_r^D, 1]$.

B Proofs

B.1 Proof of Lemma 1

We first prove that $\widehat{\sigma}^P < \widetilde{\sigma}$. Assume by contradiction that $\widehat{\sigma}^P \geq \widetilde{\sigma}$. Then,

$$\begin{aligned}\int_{\widehat{\sigma}^P}^1 \left(\overline{P}_0 - p(\widehat{\sigma}^P, s_j)\right) g\left(s_j|\widehat{\sigma}^P\right) ds_j &< \left(\overline{P}_0 - p(\widehat{\sigma}^P, \widehat{\sigma}^P)\right)\left(1 - G(\widehat{\sigma}^P|\widehat{\sigma}^P)\right) \\ &\leq 0\end{aligned}$$

which contradicts the definition of $\widehat{\sigma}^P$.

We introduce functions ψ and $\mathcal{L}$ defined by

$$\psi(x) \equiv \int_x^1 \left(\overline{P}_0 - p(x, s_j)\right) g\left(s_j|x\right) ds_j \tag{30}$$

$$\mathcal{L}\left(s|x\right) \equiv \frac{\frac{dg(s|x)}{dx}}{g\left(s|x\right)}. \tag{31}$$

We have $\widehat{\sigma}^P$ defined by $\psi(\widehat{\sigma}^P) = 0$. Function $s \mapsto \mathcal{L}(s|x)$ is increasing according to Assumption 2.

Let us prove that ψ has a unique zero.

$$\begin{aligned}\psi'(x) &= -\left(\overline{P}_0 - p(x,x)\right) g\left(x|x\right) - \int_x^1 P_1(x,s_j) g\left(s_j|x\right) ds_j \\ &\quad + \int_x^1 \left(\overline{P}_0 - p(x,s_j)\right) \mathcal{L}\left(s_j|x\right) g\left(s_j|x\right) ds_j.\end{aligned}$$

The first two terms are negative (since $\widehat{\sigma}^P < \widetilde{\sigma}$). Let us focus on the third one

$$\begin{aligned}\int_x^1 \left(\overline{P}_0 - p(x,s_j)\right) \mathcal{L}\left(s_j|x\right) g\left(s_j|x\right) ds_j &= \int_x^{\alpha(x)} \left(\overline{P}_0 - p(x,s_j)\right) \mathcal{L}\left(s_j|x\right) g\left(s_j|x\right) ds_j \\ &\quad + \int_{\alpha(x)}^1 \left(\overline{P}_0 - p(x,s_j)\right) \mathcal{L}\left(s_j|x\right) g\left(s_j|x\right) ds_j \\ &\leq \mathcal{L}\left(\alpha(x)|x\right) \int_x^{\alpha(x)} \left(\overline{P}_0 - p(x,s_j)\right) g\left(s_j|x\right) ds_j \\ &\quad + \mathcal{L}\left(\alpha(x)|x\right) \int_{\alpha(x)}^1 \left(\overline{P}_0 - p(x,s_j)\right) g\left(s_j|x\right) ds_j \\ &= \mathcal{L}\left(\alpha(x)|x\right) \psi(x)\end{aligned}$$

where the inequality comes for the fact that $\mathcal{L}$ is increasing in s and that $\overline{P}_0 - p(x,s_j) > 0 \Leftrightarrow s_j < \alpha(x)$. Therefore, the derivative of function ψ is negative when ψ equals zero, and the zero of function ψ, if it exists is unique. Assumption 1(i) implies that $\psi(0) > 0$. Moreover $\psi(1) = 0$. As a consequence, ψ is positive and then negative as x increases and $\widehat{\sigma}^P$ always exists and is unique.

B.2 Proof of Proposition 1

See Subsection A.1.

B.3 Proof of Proposition 2

It is straightforward to see that $L(x|s)$ is increasing in κ. Remember that

$$P^P(s) = \overline{P}_0(1 - L(\widehat{\sigma}^P|s)) + \int_s^{\widehat{\sigma}^P} p(x,x) dL(x|s).$$

As $\widehat{\sigma}^P$ is independent from κ, $L(\widehat{\sigma}^P|s)$ is increasing in κ as we just underlined. This also implies that $\int_s^{\widehat{\sigma}^P} p(x,x) dL(x|s)$ is increasing in κ because, for $\kappa' > \kappa$ function L_κ first order stochastic dominates $L_{\kappa'}$.[16]

The boundary between regions II_i and III_i is defined by $\left\{(s_i, s_j) \in [0,1]^2 | P^P(s_i) = p(s_i, s_j)\right\}$. For a given s_j, if κ increases, in order the equality $P^P(s_i) = p(s_i, s_j)$ to hold it must be the case that s_i decreases, do that II_i shrinks. As a consequence, III_i expands. □

B.4 Proof of Lemma 3

From the definition of $\widehat{\sigma}^P$ (equation (8)), we obtain that

$$\frac{\partial \widehat{\sigma}^P}{\partial \overline{P}_0} = -\frac{1 - G\left(\widehat{\sigma}^P | \widehat{\sigma}^P\right)}{-\left(\overline{P}_0 - p\left(\widehat{\sigma}^P, \widehat{\sigma}^P\right)\right) g\left(\widehat{\sigma}^P | \widehat{\sigma}^P\right) - \int_{\widehat{\sigma}^P}^1 p_1\left(\widehat{\sigma}^P, s\right) g\left(s | \widehat{\sigma}^P\right) ds + \int_{\widehat{\sigma}^P}^1 \left(\overline{P}_0 - p\left(\widehat{\sigma}^P, s\right)\right) g_2\left(s | \widehat{\sigma}^P\right) ds}$$

The denominator being negative, $\frac{\partial \widehat{\sigma}^P}{\partial \overline{P}_0} > 0$. □

B.5 Proof of Lemma ??

$$\frac{\partial P^P(s)}{\partial \overline{P}_0} = 1 - L\left(\widehat{\sigma}^P | s\right) - L_1\left(\widehat{\sigma}^P | s\right) \frac{\widehat{\sigma}^P}{\partial \overline{P}_0} \left(\overline{P}_0 - p\left(\widehat{\sigma}^P, \widehat{\sigma}^P\right)\right).$$

The first term is positive and corresponds to the direct effect, whereas the second term is negative and corresponds to the indirect effect.

Observing that

$$L_1\left(\widehat{\sigma}^P | s\right) = \kappa \frac{g\left(\widehat{\sigma}^P | \widehat{\sigma}^P\right)}{1 - G\left(\widehat{\sigma}^P | \widehat{\sigma}^P\right)} \left(1 - L\left(\widehat{\sigma}^P | s\right)\right),$$

we have that

$$\frac{\partial P^P(s)}{\partial \overline{P}_0} = \left(1 - L\left(\widehat{\sigma}^P | s\right)\right) \left(1 - \frac{\widehat{\sigma}^P}{\partial \overline{P}_0} \left(\overline{P}_0 - p\left(\widehat{\sigma}^P, \widehat{\sigma}^P\right)\right) \kappa \frac{g\left(\widehat{\sigma}^P | \widehat{\sigma}^P\right)}{1 - G\left(\widehat{\sigma}^P | \widehat{\sigma}^P\right)}\right).$$

The second term equals

$$\frac{-(1-\kappa)\left(\overline{P}_0 - p\left(\widehat{\sigma}^P, \widehat{\sigma}^P\right)\right) g\left(\widehat{\sigma}^P | \widehat{\sigma}^P\right) - \int_{\widehat{\sigma}^P}^1 p_1\left(\widehat{\sigma}^P, s\right) g\left(s | \widehat{\sigma}^P\right) ds + \int_{\widehat{\sigma}^P}^1 \left(\overline{P}_0 - p\left(\widehat{\sigma}^P, s\right)\right) g_2\left(s | \widehat{\sigma}^P\right) ds}{-\left(\overline{P}_0 - p\left(\widehat{\sigma}^P, \widehat{\sigma}^P\right)\right) g\left(\widehat{\sigma}^P | \widehat{\sigma}^P\right) - \int_{\widehat{\sigma}^P}^1 p_1\left(\widehat{\sigma}^P, s\right) g\left(s | \widehat{\sigma}^P\right) ds + \int_{\widehat{\sigma}^P}^1 \left(\overline{P}_0 - p\left(\widehat{\sigma}^P, s\right)\right) g_2\left(s | \widehat{\sigma}^P\right) ds}$$

In the proof of Lemma 1, we have shown that each term of this fraction is negative, so that $P^P(s)$ is an increasing function of $\overline{P}_0$. □

[16] The subscript "κ" indicates the parameter value, κ.

B.6 Proof of Lemma 2

Introduce function ϕ and θ defined by

$$\phi(x) \equiv \int_0^x \left(\overline{P}_0 - p(x,s_j)\right) g\left(s_j|x\right) ds_j \tag{32}$$

$$\theta(x) \equiv \psi(x) + (1-\kappa)\phi(x) \tag{33}$$

where ψ is defined by equation (30) in the proof of Lemma 1. $\widehat{\sigma}^D$ (equation (14)) is defined by $\theta(\widehat{\sigma}^D) = 0$. Observe that the only possibility for such an equality to be satisfied is that $\phi(\widehat{\sigma}^D) > 0$ and $\psi(\widehat{\sigma}^D) < 0$.

We also introduce function Φ and Ψ defined by

$$\Phi(x) \equiv \mathbb{E}\left[\overline{P}_0 - p(x,S)|S < x\right] \tag{34}$$

$$\Psi(x) \equiv \mathbb{E}\left[\overline{P}_0 - p(x,S)|S > x\right] \tag{35}$$

so that

$$\theta\left(x\right) = \left(1 - G\left(x|x\right)\right)\Psi\left(x\right) + \left(1-\kappa\right)G\left(x|x\right)\Phi\left(x\right).$$

We know from Milgrom and Weber (1982) that Φ and Ψ are decreasing functions. As in the proof of Lemma 1, we are going to prove that θ can cancel only once and that its derivative is negative at this point.

We also know that $\left(1 - G\left(x|x\right)\right)\Psi\left(x\right) + G\left(x|x\right)\Phi\left(x\right)$ is decreasing in x implying that

$$\frac{dG\left(x|x\right)}{dx} < -\frac{G\left(x|x\right)\Phi'(x) - \left(1 - G\left(x|x\right)\right)\Psi'(x)}{\Phi(x) - \Psi(x)}.$$

$$\theta'(x) = -(1-\kappa)G(x|x)\Phi'(x) + (1 - G(x|x))\Psi'(x) + \frac{dG\left(x|x\right)}{dx}\left((1-\kappa)\Phi(x) - \Psi(x)\right)$$

Taking into account the fact that $\theta(\widehat{\sigma}^D) = 0$, we have that

$$(1-\kappa)\Phi(\widehat{\sigma}^D) - \Psi(\widehat{\sigma}^D) = -\frac{(1 - G(\widehat{\sigma}^D|\widehat{\sigma}^D))\Psi(\widehat{\sigma}^D) + G(\widehat{\sigma}^D|\widehat{\sigma}^D)\Psi(\widehat{\sigma}^D)}{G(\widehat{\sigma}^D|\widehat{\sigma}^D)}.$$

Therefore,

$$\theta'(\widehat{\sigma}^D) = -(1-\kappa)G(\widehat{\sigma}^D|\widehat{\sigma}^D)\Phi'(\widehat{\sigma}^D) + (1 - G(x|x))\Psi'(\widehat{\sigma}^D) - \frac{\frac{dG(x|x)}{dx}\big|_{x=\widehat{\sigma}^D}}{G\left(\widehat{\sigma}^D|\widehat{\sigma}^D\right)}\Psi(\widehat{\sigma}^D).$$

The first two terms are negative and the third one is negative if $\frac{dG(x|x)}{dx}|_{x=\widehat{\sigma}^D} < 0$. To complete the proof, we must treat the case where $\frac{dG(x|x)}{dx}|_{x=\widehat{\sigma}^D} > 0$. To do that, we have to detail the expressions of $\phi'(x)$ and $\psi'(x)$.

$$\begin{aligned}
\theta'(x) &= \psi'(x) + (1-\kappa)\phi'(x) \\
&= -\left(\overline{P}_0 - p(x,x)\right) g\left(x|x\right) - \int_x^1 p_1(x,s_j) g\left(s_j|x\right) ds_j \\
&\quad + \int_x^1 \left(\overline{P}_0 - p(x,s_j)\right) g_2\left(s_j|x\right) ds_j + (1-\kappa)\left(\overline{P}_0 - p(x,x)\right) g\left(x|x\right) \\
&\quad -(1-\kappa)\int_0^x p_1(x,s_j) g\left(s_j|x\right) ds_j + (1-\kappa)\int_0^x \left(\overline{P}_0 - p(x,s_j)\right) g_2\left(s_j|x\right) ds_j \\
&= -\int_x^1 p_1(x,s_j) g\left(s_j|x\right) ds_j - (1-\kappa)\int_0^x p_1(x,s_j) g\left(s_j|x\right) ds_j - \kappa\left(\overline{P}_0 - p(x,x)\right) g\left(x|x\right) \\
&\quad + \int_x^1 \left(\overline{P}_0 - p(x,s_j)\right) g_2\left(s_j|x\right) ds_j + (1-\kappa)\int_0^x \left(\overline{P}_0 - p(x,s_j)\right) g_2\left(s_j|x\right) ds_j.
\end{aligned}$$

An integration by part of the last two terms imply that

$$\begin{aligned}
\theta'(x) &= -\int_x^1 p_1(x,s_j) g\left(s_j|x\right) ds_j - (1-\kappa)\int_0^x p_1(x,s_j) g\left(s_j|x\right) ds_j - \kappa\left(\overline{P}_0 - p(x,x)\right) g\left(x|x\right) \\
&\quad + \int_x^1 p_2(x,s_j) G_2\left(s_j|x\right) ds_j + (1-\kappa)\int_0^x p_2(x,s_j) G_2\left(s_j|x\right) ds_j - \kappa\left(\overline{P}_0 - p(x,x)\right)\left(G_1\left(x|x\right) + G_2\left(x|x\right.\right.
\end{aligned}$$

The first four terms are negative (remember that affiliation implies that $G_2 < 0$). As for the last term, it is negative when evaluated at $x = \widehat{\sigma}^D$ if $\widehat{\sigma}^D < \widetilde{\sigma}$ since $\frac{dG(x|x)}{dx}|_{x=\widehat{\sigma}^D} > 0$.

Therefore the derivative of function θ is negative when θ equals zero, and the zero of function θ, if it exists and is smaller than $\widetilde{\sigma}$ is unique. Assumption 1(i) implies that $\theta(0) > 0$ and $\theta(1) < 0$. This implies that θ is positive and then negative as x increases and that $\widehat{\sigma}^D < \widetilde{\sigma}$ always exists and is unique when $\theta(\widetilde{\sigma}) < 0$. □

B.7 Proof of Lemma 3

We prove first that if the separating equilibrium does not exist ($\widetilde{\sigma}^D > \widetilde{\sigma}$), then the semi-pooling equilibrium exists and is unique.

The first part of this proof goes through a series of steps. Let us first introduce function

I, J and K

$$\begin{aligned} I(x,y) &= \int_x^y \left(\overline{P}_0 - p(x,t)\right) g(t|x)dt && (36)\\ J(x,y) &= \int_x^y \left(\overline{P}_0 - p(y,t)\right) g(t|y)dt && (37)\\ H(x,y) &= \theta(y) + \frac{\kappa}{2} J(x,y) && (38)\end{aligned}$$

where θ is defined by equation (33). Observe that $\psi(x) = I(x,1)$, $\phi(x) = J(0,x)$ and $\theta(x) = I(x,1) + (1-\kappa)J(0,x)$.

Step 1: We show that $\alpha(\overline{\sigma}^D) \le \underline{\sigma}^D < \tilde{\sigma} < \alpha(\underline{\sigma}^D) \le \overline{\sigma}^D \le \hat{\sigma}^D$.

To prove this step, assume that $(\underline{\sigma}^D, \overline{\sigma}^D)$ is a solution meaning that $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \overline{\sigma}^D) = 0$. $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$ implies that

$$\begin{aligned} \overline{P}_0 \left(G(\overline{\sigma}^D|\underline{\sigma}^D) - G(\underline{\sigma}^D|\underline{\sigma}^D)\right) &= \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} p(\underline{\sigma}^D, t) g(t|\underline{\sigma}^D)dt \\ &> p(\underline{\sigma}^D, \underline{\sigma}^D) \left(G(\overline{\sigma}^D|\underline{\sigma}^D) - G(\underline{\sigma}^D|\underline{\sigma}^D)\right). \end{aligned}$$

Therefore, $\overline{P}_0 > p(\underline{\sigma}^D, \underline{\sigma}^D)$ implying that $\underline{\sigma}^D < \tilde{\sigma}$.

Remember that $t \mapsto \overline{P}_0 - p(\underline{\sigma}^D, t)$ is a decreasing function. In order to have $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$, it must be the case that $\overline{P}_0 - p(\underline{\sigma}^D, t)$ is first positive and then negative as t increases from $\underline{\sigma}^D$ to $\overline{\sigma}^D$. In particular, we must have that $\overline{P}_0 - p(\underline{\sigma}^D, \overline{\sigma}^D) < 0$. This implies that $\overline{\sigma}^D > \alpha(\underline{\sigma}^D)$ and $\underline{\sigma}^D > \alpha(\overline{\sigma}^D)$ where function α is defined by equation (5). Note that the symmetry of p and the fact that it is increasing with respect to each of its argument imply that $\alpha = \alpha^{-1}$.

To prove that $\overline{\sigma}^D \le \hat{\sigma}^D$, we show that $J(\underline{\sigma}^D, \overline{\sigma}^D) < 0$ so that $\theta(\overline{\sigma}^D) > 0$.

$$\begin{aligned} J(\underline{\sigma}^D, \overline{\sigma}^D) &= \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P}_0 - p(t, \overline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\ &\le \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P}_0 - p(t, \underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\ &= \int_{\underline{\sigma}^D}^{\alpha(\underline{\sigma}^D)} \left(\overline{P}_0 - p(t, \underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt + \int_{\alpha(\underline{\sigma}^D)}^{\overline{\sigma}^D} \left(\overline{P}_0 - p(t, \underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\ &\le \frac{g(\alpha(\underline{\sigma}^D)|\overline{\sigma}^D)}{g(\alpha(\underline{\sigma}^D)|\underline{\sigma}^D)} I(\underline{\sigma}^D, \overline{\sigma}^D) \\ &= 0. \end{aligned}$$

The second inequality holds because $t \mapsto \frac{g(t|y)}{g(t|x)}$ is an increasing function $\forall x \le y$.

It therefore holds that $\alpha(\overline{\sigma}^D) \le \underline{\sigma}^D \le \tilde{\sigma} \le \alpha(\underline{\sigma}^D) \le \overline{\sigma}^D \le \hat{\sigma}^D$. This allows us to define the region $\mathcal{D} \equiv \left\{(x,y) \in [0,1]^2 | \alpha(y) \leq x \leq \widetilde{\sigma} \leq \alpha(x) \leq y \leq \widehat{\sigma}^D\right\}$ to which the solution to the following system should belong to

$$\begin{cases} I(x,y) & = & 0 \\ H(x,y) & = & 0. \end{cases}$$

Step 2: We show that $x_I(y)$ defined by $I(x_I(y),y) = 0$ on $\mathcal{D}_y = \left\{\widetilde{\sigma} \leq y \leq \widehat{\sigma}^D | \alpha(y) \leq x_I(y)\right\}$ is a decreasing function.

The implicit function theorem implies that

$$\frac{dx_I(y)}{dy} = -\frac{I_1(x_I(y),y)}{I_2(x_I(y),y)}.$$

$I_2(x_I(y),y) = (\overline{P}_0 - p(x_I(y),y))g(y|x_I(y)) \leq 0$ since $\alpha(y) \leq x_I(y)$ (or equivalently $y \geq \alpha(x_I(y)))$.

$$\begin{aligned} I_1(x_I(y),y) &= -\left(\overline{P}_0 - p(x_I(y),x_I(y))\right) g(x_I(y)|x_I(y)) - \int_{x_I(y)}^{y} P_1(x_I(y),t)g(t|x_I(y)) \\ &\quad + \int_{x_I(y)}^{y} (\overline{P}_0 - p(x_I(y),t))\mathcal{L}\left(t|x_I(y)\right) g(t|x_I(y))dt \end{aligned}$$

The first two terms are negative (the first because $x_I(y) \leq \widetilde{\sigma}$). As for the third term, using the fact that $t \mapsto \mathcal{L}\left(t|x_I(y)\right)$ is an increasing function and as we did in the proofs of Lemmas 1 and 2

$$\begin{aligned} & \int_{x_I(y)}^{y} (\overline{P}_0 - p(x_I(y),t))\mathcal{L}\left(t|x_I(y)\right) g(t|x_I(y))dt \\ \leq\ & \mathcal{L}\left(\alpha(x_I(y))|x_I(y)\right) \int_{x_I(y)}^{y} (\overline{P}_0 - p(x_I(y),t))g(t|x_I(y))dt \\ =\ & 0. \end{aligned}$$

This implies that $y \mapsto x_I(y)$ is a decreasing function.

Note moreover that $x_I(\widetilde{\sigma}) = \widetilde{\sigma}$ and that $x_I(\widehat{\sigma}^D) > \alpha(\widehat{\sigma}^D)$. To prove this last inequality, assume by contradiction that $x_I(\widehat{\sigma}^D) \leq \alpha(\widehat{\sigma}^D)$. Knowing that $I(x_I(y),y) = 0$ and since $p(x_I(\widehat{\sigma}^D),t) < p(\alpha(\widehat{\sigma}^D),t)$ this implies that

$$\int_{x_I(\widehat{\sigma}^D)}^{\widehat{\sigma}^D} \left(\overline{P}_0 - p(\alpha(\widehat{\sigma}^D),t)\right) g(t|x_I(\widehat{\sigma}^D))dt < 0.$$

However, as $p(\alpha(\widehat{\sigma}^D), t) < p(\alpha(\widehat{\sigma}^D), \widehat{\sigma}^D) = \overline{P}_0$,

$$\int_{x_I(\widehat{\sigma}^D)}^{\widehat{\sigma}^D} \left(\overline{P}_0 - p(\alpha(\widehat{\sigma}^D), t)\right) g(t|x_I(\widehat{\sigma}^D))dt > 0,$$

hence a contradiction implying that $x_I(\widehat{\sigma}^D) > \alpha(\widehat{\sigma}^D)$.

The last property that remains to be showed for this function x_I is that $y \mapsto x_I(y)$ and $y \mapsto \alpha(y)$ only cross once when $y \in [\widetilde{\sigma}, \widehat{\sigma}^D]$. This is not a priori obvious since the two functions are decreasing. We know that $x_I(\widetilde{\sigma}) = \alpha(\widetilde{\sigma}) = \widetilde{\sigma}$. Assume that there exists $\overline{y} \in (\widetilde{\sigma}, \widehat{\sigma}^D]$ such that $x_I(\overline{y}) = \alpha(\overline{y})$. By definition of x_I, this implies that

$$\int_{\alpha(\overline{y})}^{\overline{y}} \left(\overline{P}_0 - p\left(\alpha(\overline{y}), t\right) g\left(t|\alpha(\overline{y})\right)\right) dt = 0.$$

However, $p\left(\alpha(\overline{y}), t\right) < p\left(\alpha(\overline{y}), \overline{y}\right) = \overline{P}_0$, $\forall t \in [\alpha(\overline{y}), \overline{y}]$, so that it is not possible that the integral equals 0. Therefore such an $\overline{y}$ does not exist. As a consequence, $y \mapsto x_I(y)$ and $y \mapsto \alpha(y)$ only cross for $y = \widetilde{\sigma}$, and $\forall y \in [\widetilde{\sigma}, \widehat{\sigma}^D]$, $x_I(y) > \alpha(y)$.

Step 3: we show tat $y_H(x)$ defined by $H(x, y_H(x)) = 0$ on $\mathcal{D}_x = \left\{\alpha\left(\widehat{\sigma}^D\right) \leq x \leq \widetilde{\sigma} | \alpha(x) \leq y_H(x)\right\}$ is an increasing function.
The implicit function theorem implies that

$$\frac{dy_H(x)}{dx} = -\frac{H_1(x, y_H(x))}{H_2(x, y_H(x))}.$$

Remembering that $H(x, y) = \theta(y) + (\kappa/2)J(x, y)$ (where θ is defined by equation (33)), this reads

$$\frac{dy_H(x)}{dx} = -\frac{\frac{\kappa}{2}J_1(x, y_H(x))}{\theta'(y_H(x)) + \frac{\kappa}{2}J_2(x, y_H(x))}.$$

$$\begin{aligned} J_1(x, y_H(x)) &= -(\overline{P}_0 - p(x, y_H(x)))g(x|y_H(x))) \\ &> -(\overline{P}_0 - p(x, \alpha(x)))g(x|y_H(x))) \\ &= 0. \end{aligned}$$

$$
\begin{aligned}
H_2(x,y_H(x)) &= -\left(\overline{P}_0 - p\left(y_H(x),y_H(x)\right)\right) g\left(y_H(x)|y_H(x)\right) - \int_{y_H(x)}^{1} P_1\left(y_H(x),t\right) g\left(t|y_H(x)\right) dt \\
&\quad + \int_{y_H(x)}^{1} \left(\overline{P}_0 - p\left(y_H(x),t\right) g\left(t|y_H(x)\right)\right) \mathcal{L}\left(t|y_H(x)\right) dt \\
&\quad + (1-\kappa)\left(\left(\overline{P}_0 - p\left(y_H(x),y_H(x)\right)\right) g\left(y_H(x)|y_H(x)\right) - \int_0^{y_H(x)} P_2\left(t,y_H(x)\right) g\left(t,|y_H(x)\right) dt\right. \\
&\quad \left. + \int_0^{y_H(x)} \left(\overline{P}_0 - p\left(t,y_H(x)\right)\right) g\left(t|y_H(x)\right) \mathcal{L}\left(t|y_H(x)\right) dt\right) \\
&\quad + \frac{\kappa}{2}\left(\left(\overline{P}_0 - p\left(y_H(x),y_H(x)\right)\right) g\left(y_H(x),y_H(x)\right) - \int_x^{y_H(x)} P_2\left(t,y_H(x)\right) g\left(t|y_H(x)\right) dt\right. \\
&\quad \left. + \int_x^{y_H(x)} \left(\overline{P}_0 - p\left(t,y_H(x)\right)\right) g\left(t|y_H(x)\right) \mathcal{L}\left(t|y_H(x)\right) dt\right) \\
&= -\frac{\kappa}{2}\left(\overline{P}_0 - p\left(y_H(x),y_H(x)\right)\right) g\left(y_H(x)|y_H(x)\right) - \int_{y_H(x)}^{1} P_1\left(y_H(x),t\right) g\left(t|y_H(x)\right) dt \\
&\quad - (1-\kappa)\int_0^{y_H(x)} P_2\left(t,y_H(x)\right) g\left(t,|y_H(x)\right) dt - \frac{\kappa}{2}\int_x^{y_H(x)} P_2\left(t,y_H(x)\right) g\left(t|y_H(x)\right) dt \\
&\quad + \int_{y_H(x)}^{1} \left(\overline{P}_0 - p\left(y_H(x),t\right) g\left(t|y_H(x)\right)\right) \mathcal{L}\left(t|y_H(x)\right) dt \\
&\quad + (1-\kappa)\int_0^{y_H(x)} \left(\overline{P}_0 - p\left(t,y_H(x)\right)\right) g\left(t|y_H(x)\right) \mathcal{L}\left(t|y_H(x)\right) dt \\
&\quad + \frac{\kappa}{2}\int_x^{y_H(x)} \left(\overline{P}_0 - p\left(t,y_H(x)\right)\right) g\left(t|y_H(x)\right) \mathcal{L}\left(t|y_H(x)\right) dt.
\end{aligned}
$$

The first four terms are negative. Let us analyze the last three terms.

$$
\begin{aligned}
& \int_{y_H(x)}^{1} \left(\overline{P}_0 - p\left(y_H(x),t\right)\right) g\left(t|y_H(x)\right) \mathcal{L}\left(t|y_H(x)\right) dt \\
& + (1-\kappa)\int_0^{y_H(x)} \left(\overline{P}_0 - p\left(t,y_H(x)\right)\right) g\left(t|y_H(x)\right) \mathcal{L}\left(t|y_H(x)\right) dt \\
& + \frac{\kappa}{2}\int_x^{y_H(x)} \left(\overline{P}_0 - p\left(t,y_H(x)\right)\right) g\left(t|y_H(x)\right) \mathcal{L}\left(t|y_H(x)\right) dt \\
\leq\ & \mathcal{L}\left(y_H(x)|y_H(x)\right) I\left(y_H(x),1\right) + (1-\kappa)\mathcal{L}\left(\alpha\left(y_H(x)\right)|y_H(x)\right) J\left(0,y_H(x)\right) + \frac{\kappa}{2}\mathcal{L}\left(x|y_H(x)\right) J\left(0,y_H(x)\right) \\
\leq\ & \mathcal{L}\left(\alpha\left(y_H(x)\right)|y_H(x)\right)\left(I\left(y_H(x),1\right) + (1-\kappa)J\left(0,y_H(x)\right) + \frac{\kappa}{2}J\left(0,y_H(x)\right)\right) \\
=\ & \mathcal{L}\left(\alpha\left(y_H(x)\right)|y_H(x)\right) H(x,y_H(x)).
\end{aligned}
$$

The first inequality holds because

- the first integral is negative. Indeed, $\widetilde{\sigma} \leq y_H(x) \leq t$ so that $\overline{P}_0 - p\left(y_H(x),t\right) \leq 0$,

$\forall t \in [y_H(x), 1]$;

- the second integral is positive (resp. negative) when $t \in [0, \alpha(y_H(x))]$ (resp. $t \in [\alpha(y_H(x)), y_H(x)]$);

- the third integral is negative. Indeed, $\alpha(y_H(x)) \leq x \leq t$ so that $\overline{P}_0 - p(y_H(x), t) \leq 0$, $\forall t \in [x, y_H(x)]$;

- function $t \mapsto \mathcal{L}(t|y_H(x))$ is increasing (Assumption 2).

The second inequality holds because $\alpha(y_H(x)) \leq x \leq y_H(x)$ and because function $t \mapsto \mathcal{L}(t|y_H(x))$ is increasing. This implies that $H_2(x, y_H(x)) \leq 0$ and therefore y_H is increasing function.

Moreover note that $y_H(\widetilde{\sigma}) \in [\widetilde{\sigma}, \widehat{\sigma}^D]$. Suppose by contradiction that $y_H(\widetilde{\sigma}) > \widehat{\sigma}^D$. In this case, $\theta(y_H(\widetilde{\sigma})) < 0$ and $J(\widetilde{\sigma}), y_H(\widetilde{\sigma}) > 0$. This implies that

$$\begin{aligned} 0 &< \int_{\widetilde{\sigma}}^{y_H(\widetilde{\sigma})} \left(\overline{P}_0 - p(y_H(\widetilde{\sigma}), t)\right) g(t|y_H(\widetilde{\sigma}))\, dt \\ &< \int_{\widetilde{\sigma}}^{y_H(\widetilde{\sigma})} \left(\overline{P}_0 - p(\widetilde{\sigma}, t)\right) g(t|y_H(\widetilde{\sigma}))\, dt \\ &< 0, \end{aligned}$$

leading to a contradiction. Therefore $y_H(\widetilde{\sigma}) \leq \widehat{\sigma}^D$. The same reasoning implies that $y_H(\widetilde{\sigma}) \leq \widetilde{\sigma}$.

Step 4: We show that the solution $(\underline{\sigma}^D, \overline{\sigma}^D)$ is unique.
x_I is a deceasing function such that $x_I(\widetilde{\sigma}) = \widetilde{\sigma}$, $x_I(\widehat{\sigma}^D) = \widetilde{\sigma}$ and Y_H is an increasing function such that $y_H(\widetilde{\sigma}) \in [\widetilde{\sigma}, \widehat{\sigma}^D]$. As $x_I(y) > \alpha(y)$, $\forall y \in (\widetilde{\sigma}, \widehat{\sigma}^D]$, the two function cross only once on $\mathcal{D}$. This intersection point that is unique corresponds to the unique solution of the system that is $(\underline{\sigma}^D, \overline{\sigma}^D)$.

The last part of the proof consists in showing that if the semi-pooling equilibrium exists, then the separating equilibrium does not exist. We are going to prove that if the separating equilibrium exists then the semi-pooling equilibrium does not exist. Assume therefore that $\widehat{\sigma}^P < \widetilde{\sigma}$. We have proven in Step 1 that $\overline{\sigma}^P \leq \widehat{\sigma}^P$, this implies that $\underline{\sigma}^D < \overline{\sigma}^D < \widetilde{\sigma}$. But in this case, $I(\underline{\sigma}^D, \overline{\sigma}^D) > 0$. Therefore, if $\widehat{\sigma}^P < \widetilde{\sigma}$, there do not exist $(\underline{\sigma}^D, \overline{\sigma}^D)$ satisfying $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \overline{\sigma}^D) = 0$.

It remains to prove that

$$\int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P}_0 - p(s,t)\right) g\left(t|s\right) dt \leq 0 \forall s \in [\underline{\sigma}^D \overline{\sigma}^D].$$

$$\begin{aligned}
\int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P}_0 - p(s,t)\right) g\left(t|s\right) dt &= \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P}_0 - p(t,s)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\
&\leq \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P}_0 - p(t,\underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\
&= \int_{\underline{\sigma}^D}^{\alpha(\underline{\sigma}^D)} \left(\overline{P}_0 - p(t,\underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\
&\quad + \int_{\alpha(\underline{\sigma}^D)}^{\overline{\sigma}^D} \left(\overline{P}_0 - p(t,\underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\
&\leq \frac{g(\alpha(\underline{\sigma}^D)|s)}{g(\alpha(\underline{\sigma}^D)|\underline{\sigma}^D)} I(\underline{\sigma}^D, \overline{\sigma}^D) \\
&= 0.
\end{aligned}$$

B.8 Proof of Proposition 5

See Subsection A.2.

B.9 Proof of Lemma 4

$\theta(\widehat{\sigma}^D) = \psi(\widehat{\sigma}^D) + (1-\kappa)\,\phi(\widehat{\sigma}^D) = 0$. As we noted in the proof of Lemma 2, $\phi(\widehat{\sigma}^D) > 0$.

The implicit function theorem implies that

$$\frac{\partial \widehat{\sigma}^D}{\partial \kappa} = \frac{\phi\left(\widehat{\sigma}^D\right)}{\psi'\left(\widehat{\sigma}^D\right) + (1-\kappa)\,\phi'\left(\widehat{\sigma}^D\right)} = \frac{\phi\left(\widehat{\sigma}^D\right)}{\theta'\left(\widehat{\sigma}^D\right)}.$$

In Lemma 2, we have proved that $\theta'(\widehat{\sigma}^D) \leq 0$ so that $\frac{\partial \widehat{\sigma}^D}{\partial \kappa} \leq 0$.

If $\widehat{\sigma}^D > \widetilde{\sigma}$, $\underline{\sigma}^D$ and $\overline{\sigma}^D$ are such that

$$\begin{cases} I(\underline{\sigma}^D, \overline{\sigma}^D) &= 0 \\ H(\underline{\sigma}^D, \overline{\sigma}^D) &= 0. \end{cases}$$

The implicit function theorem implies that

$$\begin{aligned}\frac{\partial \overline{\sigma}^D}{\partial \kappa} &= \frac{J(0,\overline{\sigma}^D)-\frac{1}{2}J(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\overline{\sigma}^D,1)+(1-\kappa)J_1(0,\overline{\sigma}^D)+\frac{\kappa}{2}J_2(\underline{\sigma}^D,\overline{\sigma}^D)-\frac{\kappa}{2}J_1(\underline{\sigma}^D,\overline{\sigma}^D)\frac{I_2(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}}\\ \frac{\partial \underline{\sigma}^D}{\partial \kappa} &= -\frac{\partial \overline{\sigma}^D}{\partial \kappa}\frac{I_2(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}.\end{aligned}$$

We have proven in Step 3 of the proof of Lemma 3 that $H_2(\underline{\sigma}^D,\overline{\sigma}^D)=I_1(\overline{\sigma}^D,1)+(1-\kappa)J_1(0,\overline{\sigma}^D)+\frac{\kappa}{2}J_2(\underline{\sigma}^D,\overline{\sigma}^D)\leq 0$ and that $J_1(\underline{\sigma}^D,\overline{\sigma}^D)\geq 0$. In Step 2 of the proof of Lemma 3, we also showed that $\frac{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}{I_2(\underline{\sigma}^D,\overline{\sigma}^D)}\geq 0$. This implies that the denominator of $\frac{\partial \overline{\sigma}^D}{\partial \kappa}$ is negative and that $\frac{\partial \overline{\sigma}^D}{\partial \kappa}$ and $\frac{\partial \underline{\sigma}^D}{\partial \kappa}$ have opposite signs.

$$\begin{aligned}J(0,\overline{\sigma}^D)-\frac{1}{2}J(\underline{\sigma}^D,\overline{\sigma}^D) &= \frac{1}{\kappa}\left(I(\overline{\sigma}^D,1)+J(0,\overline{\sigma}^D)\right).\\ &= \frac{1}{\kappa}\left(I(\overline{\sigma}^D,1)+(1-\kappa)J(0,\overline{\sigma}^D)+\kappa J(0,\overline{\sigma}^D)\right).\end{aligned}$$

In Step 1 of the proof of Lemma 3, we also proved that $J(\underline{\sigma}^D,\overline{\sigma}^D)\leq 0$. Together with $H(\underline{\sigma}^D,\overline{\sigma}^D)=0$ implies that

$$I(\overline{\sigma}^D,1)+(1-\kappa)J(0,\overline{\sigma}^D)=-\frac{\kappa}{2}J(\underline{\sigma}^D,\overline{\sigma}^D)\geq 0.$$

Moreover, as $I(\overline{\sigma}^D,1)\leq 0$ and $\kappa\in(0,1)$, it must be the case that $J(0,\overline{\sigma}^D)\geq 0$. It follows that $J(0,\overline{\sigma}^D)-\frac{1}{2}J(\underline{\sigma}^D,\overline{\sigma}^D)\geq 0$. As a consequence, $\frac{\partial \overline{\sigma}^D}{\partial \kappa}\leq 0$ and $\frac{\partial \underline{\sigma}^D}{\partial \kappa}\geq 0$.

B.10 Proof of Proposition 6

Remember that the separating equilibrium exists if and only if $\widehat{\sigma}^D<\widetilde{\sigma}$. Let us analyze the function $\kappa\mapsto\widehat{\sigma}^D-\widetilde{\sigma}$. Thanks to Lemma 4, we know that this function is decreasing ($\widetilde{\sigma}$ is independent of κ). When $\kappa=1$, observe that $\widehat{\sigma}^D=\widehat{\sigma}^P$, so that $\widehat{\sigma}^D-\widetilde{\sigma}=\widehat{\sigma}^P-\widetilde{\sigma}\leq 0$. If, when $\kappa=0$, $\widehat{\sigma}^P-\widetilde{\sigma}<0$, then $\forall\kappa\in[0,1]$, $\widehat{\sigma}^D<\widetilde{\sigma}$ and the separating equilibrium always exists. If, on the contrary, when $\kappa=0$, $\widehat{\sigma}^P-\widetilde{\sigma}>0$, then there exists a unique κ^* such that the separating equilibrium (resp. semi-pooling equilibrium) exists if and only if $\kappa\geq\kappa^*$ (resp. $\kappa<\kappa^*$). The rest of the proof consists in proving that the comparison of $\widehat{\sigma}^P$ to $\widetilde{\sigma}$ when $\kappa=0$ comes down to comparing $\overline{P}_0$ to $\int_0^1 p(\widetilde{\sigma},s)g(s|\widetilde{\sigma})ds=\mathbb{E}[p(S_i,S_j)|S_j=\widetilde{\sigma}]$.

When $\kappa=0$, $\widehat{\sigma}^D$ is implicitly defined by

$$\int_0^1\left(\overline{P}_0-p(\widehat{\sigma}^D,t)\right)g(t|\widehat{\sigma}^D)dt=0.$$

Let us introduce function Λ defined by $\Lambda(x) = \int_0^1 \left(\overline{P}_0 - p(x,t)\right) g(t|x)dt$. As in the prof of Lemmas 1 and 2, we can prove that

$$\begin{aligned}\Lambda'(x) &= \int_0^1 -P_1(x,t)g(t|x)dt + \int_0^1 \left(\overline{P}_0 - p(x,t)\right) \mathcal{L}(t|x)g(t|x)dt \\ &\leq \int_0^1 -P_1(x,t)g(t|x)dt + \mathcal{L}(\alpha(x)|x)\Lambda(x)\end{aligned}$$

so that $\Lambda'(\widehat{\sigma}^D) \leq 0$. Because of Assumption 1(i), $\Lambda(0) > 0$ and $\Lambda(1) < 0$, so that Λ is positive if and only if $x \leq \widehat{\sigma}^D$. It follows that $\widehat{\sigma}^D > \widetilde{\sigma}$ if and only if $\Lambda(\widetilde{\sigma}) > 0$ which comes down to the condition stated in the proposition, that is $\overline{P}_0 > \int_0^1 p(\widetilde{\sigma}, s)g(s|\widetilde{\sigma})ds$.

B.11 Proof of Lemma ??

Cf proof of Lemmas 3 and **??**.

B.12 Proof of Lemma 5

Remember that

- $\widehat{\sigma}^P$ is such that $\psi(\widehat{\sigma}^P) = 0$ where ψ is defined in Equation (30),
- $\widehat{\sigma}^D$ is such that $\theta(\widehat{\sigma}^D) = \psi(\widehat{\sigma}^D) + (1-\kappa)\phi(\widehat{\sigma}^D) = 0$ where ϕ and θ are defined in Equations (32) and (33),
- $\underline{\sigma}^D$ and $\overline{\sigma}^D$ are the solution of the system $I(\underline{\sigma}^D, \underline{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \underline{\sigma}^D) = 0$ where I and J are defined in Equations (37) and (38).

Assume first that $\widehat{\sigma}^D \leq \widetilde{\sigma}$.
We already noted (see the proof of Lemma 2) that $\psi(\widehat{\sigma}^D) < 0$. In addition, we also proved in Lemma 1 that $\psi(x) > 0 \Leftrightarrow x < \widehat{\sigma}^P$. As $\psi(\widehat{\sigma}^P) = 0$, this implies that $\widehat{\sigma}^P \leq \widehat{\sigma}^D$.

Assume now that $\widehat{\sigma}^D > \widetilde{\sigma}$.
$\psi(\underline{\sigma}^D) = I(\underline{\sigma}^D, 1) < I(\underline{\sigma}^D, \underline{\sigma}^D) = 0$. The same reasoning implies that $\underline{\sigma}^P \leq \widehat{\sigma}^D$.

B.13 Proof of Lemma 6

Note first that $P^P(\widehat{\sigma}^P) = \overline{P}_0 > P^D(\widehat{\sigma}^P)$. Second, imagine that the two functions P^P and P^D cross at some point such that $P^P(s) = P^D(s)$ at this point. Using the differential equations satisfied by the two premiums (Equations (9) and (15)), we have that $P^{P'}(s) > P^{D'}(s)$. Therefore, if the two functions cross, it happens only once. If $P^D(0) \leq P^P(0)$, the two curves never crossed. □

B.14 Proof of Lemma 7

B.15 Proof of Lemma 8

Since the two premiums satisfy the same differential equation, the difference between them only comes from the threshold that determine the initial conditions of the differential equation. Assume the threshold is x and let us differentiate the premium with respect to this threshold. It equals

$$-\left(\overline{P}_0 - p\left(x, x\right)\right) \frac{\partial K\left(x|s\right)}{\partial x} < 0$$

since the threshold is smaller than $\widetilde{\sigma}$ (both in the separating and in the pooling equilibrium).

If the parameters are such that the equilibrium is separating in the two cases: $\widehat{\sigma}_r^D < \widehat{\sigma}^D < \widetilde{\sigma}$, then

$$P^D(s) < P_r^D(s) \; \forall \; s \leq \widehat{\sigma}_r^D.$$

When one of the equilibrium is separating, we should check whether $\widehat{\sigma}_r^D < \underline{\sigma}^D$ and $\underline{\sigma}_r^D < \underline{\sigma}^D$. **TO BE DONE**. □

B.16 Proof of Lemma ??

By the definition of the two functions $\overline{s}_j^P(s_i)$ and $\overline{s}_j^{D_r}(s_i)$, we have that

$$\begin{aligned} & P^P(s_i) > P_r^D(s_i) \\ \Leftrightarrow \; & p\left(s_i, \overline{s}_j^P(s_i)\right) > p\left(s_i, \overline{s}_j^{D_r}(s_i)\right) \\ \Leftrightarrow \; & \overline{s}_j^P(s_i) > \overline{s}_j^{D_r}(s_i) \end{aligned}$$

□

References

[1] Back, K. and J. Zender (1993), “Auction of divisible goods : on the rationale for the treasury experiment”, *Review of Financial Studies*, 6 pp 733-764.

[2] Berger, L.A., Cummins, J.D. and S. Tennyson (1992), “Reinsurance and the liability insurance crisis”, *Journal of risk and Uncertainty,* 5 pp 253-272.

[3] Doherty, N., A. and G. Dionne (1993), “Insurance with undiversifiable risk: Contract structure and organizational form of insurance firm”, *Journal of risk and uncertainty*, 6 pp 187-203.

[4] European Commission, (2010), Commission Regulation (EU) No 267/2010 of 24 March 2010 on the application of Article 101(3) of the Treaty on the Functioning of the European Union to certain categories of agreements, decisions and concerted practices in the insurance sector

[5] Fagart, M.C., Fombaron, N. and M. Jeleva (2002), “Risk Mutualization and Competition in Insurance Markets”, *THe Geneva Papers on Risk and Insurance Theory*, 27 pp 115-141.

[6] Faure, M. and R. Van den Bergh (1995), “Restrictions of competition on insurance markets and the applicability of EC antitrust law”, *KYKLOS,* 48 pp 65-85.

[7] Fudenberg, D. and J. Tirole (1996), Game Theory, *MIT Press.*

[8] Haile, P. A. (2000), “Partial pooling at the reserve price in auctions with resale opportunities”, *Games and Economic Behavior*, 33(2), 231-248.

[9] Hendricks, K., Porter, R., and Tan, G. (2008), “Bidding rings and the winner’s curse”, *RAND Journal of Economics*, 39(4), 1018-1041.

[10] Kreps, D. M. and J. A. Scheinkmann (1983), “Quantity precommitment and Bertrand competition yield Cournot outcomes”, *Bell Journal of Economics*, 14 pp 326-337.

[11] Mahul, O. and B. D. Wright (2003), “Efficient risk sharing within a catastrophe insurance pool”, ?

[12] Milgrom, P. R., and Weber, R. J. (1982), “A theory of auctions and competitive bidding”, *Econometrica*; 50(5), 1089-1122.

[13] Wilson, R. (1979), "Auctions of shares", *Quarterly Journal of Economics*, 93 pp 675-689.