Equilibrium in Incomplete Markets with Numeraire Assets and Differential Information: An Existence Proof

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Equilibrium in incomplete markets with numeraire assets and differential information: an existence proof

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Abstract

The paper extends to asymmetric information Geanakoplos-Polemarchakis' (1986) existence theorem for incomplete financial markets with numeraire assets. It builds on a generic existence property of equilibria with real assets and differential information, and applies an asymptotic argument. It presents a two-period pure-exchange economy, with an ex ante uncertainty over the state of nature to be revealed at the second period. Asymmetric information is represented by private sets of states, that each agent is correctly informed to contain the realizable states. Consumers exchange commodities, on spot markets, and securities, on financial markets, which pay off in the same bundle of goods, conditionally on the state of nature to be revealed. Consumers have ordered smooth preferences over consumptions and a perfect foresight of future prices, along Radner (1972). With a different technique of proof, the paper also extends to numeraire assets De Boisdeffre's (2007) existence theorem for nominal assets, which characterizes existence by a no-arbitrage condition.

Key words: sequential equilibrium, temporary equilibrium, perfect foresight, existence, rational expectations, financial markets, asymmetric information, arbitrage.

JEL Classification: D52

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1 Introduction

The paper demonstrates, with new non-standard arguments, the full existence of equilibria in incomplete financial markets with numeraire assets and differential information. It thus extends Geanakoplos-Polemarchakis' (1986) existence result of symmetric information. The proof uses the generic existence property of equilibria, with real assets and differential information, that we propose in a companion paper [5]. We build a converging sequence of such equilibria in auxiliary economies with real assets, and show that its limit is an equilibrium of our initial economy.

This economy is one of pure exchange, with two periods and an ex ante uncertainty over the state of nature to prevail. Asymmetric information is represented by private finite sets of states, providing the correct signals that the true state will be in those sets. There are spot markets, for goods, and financial markets, for securities, which all pay off in the same bundle of goods, called "numeraire". Consumers have endowments in goods in every expected state, ordered smooth preferences, and a perfect foresight of future prices, along Radner (1972).

When assets pay off in goods, equilibrium needs not exist, as shown by Hart (1975) in the symmetric information case. However, in [5], we show that the generic existence of equilibria is guaranteed for any arbitrage-free collection of securities and private information sets. On purely financial markets, De Boisdeffre (2007) shows that the existence of equilibria is characterized by the absence of arbitrage opportunity. For any collection of assets and information signals, De Boisdeffre (2016) shows that the latter no-arbitrage condition may be achieved, with agents having no price model, from their observing available transfers on financial markets. The current paper extends De Boisdeffre's (2007) existence characterization to numeraire assets.

The paper is organized as follows: Section 2 presents the model, the consumer's behaviour and the concept of equilibrium. Section 3 states and proves the existence theorem. An Appendix proves a technical Lemma.

2 The model

Throughout, we consider a pure-exchange economy with two periods, $t \in \{0, 1\}$, and an uncertainty, at $t = 0$, on the state of nature to prevail, at $t = 1$. There are spots markets, for goods, and incomplete financial markets, where numeraire assets are exchanged. The sets, I, S, L and J, respectively, of consumers, states of nature, commodities and assets are all finite. The non-random state of the first period $(t = 0)$ is denoted by $s = 0$ and we let $\Sigma' := \{0\} \cup \Sigma$, for every subset, Σ, of S.

We present information signals and markets, in sub-Section 2.1, consumer's behaviour and equilibria, in sub-Section 2.2, the model's notations in sub-Section 2.3.

2.1 Markets and information

Agents consume or exchange the consumption goods, $l \in L$, on both periods' spot markets. At $t = 0$, each agent, $i \in I$, receives a private information signal, $S_i \subset S$, informing her correctly that the true state will be in S_i. We assume costlessly that $S = \cup_{i \in I} S_i$ and henceforth denote $\underline{\mathbf{S}} := \cap_{i \in I} S_i$.

Future spot prices are restricted to the unit hemisphere, $\Delta := \{p \in \mathbb{R}^L_{++} : \|p\| = 1\}$. The bound one in Δ is chosen for convenience and could be replaced by any positive value in any state. Each agent, $i \in I$, forms an idiosyncratic anticipation, $p_i := (p_{is}) \in \mathbb{R}^{S_i \backslash \underline{\mathbf{S}}}_{++}$, of the spot prices in each unrealizable state, if $S_i \neq \underline{\mathbf{S}}$. For convenience, but clearly w.l.o.g., we will assume that $p_{is} = p_{js} := \overline{p}_s \in \Delta$, for every pair, $(i, j) \in I^2$, such that $s \in S_i \cap S_j \backslash \underline{\mathbf{S}}$. So, admissible commodity prices will be restricted w.l.o.g. to the set $P := \mathbb{R}^L_{++} \times \{ \ p := (p_s) \in \Delta^S : \ p_s = \overline{p}_s, \ \forall s \in S \backslash \underline{\mathbf{S}} \ \}$.

Agents may operate transfers across states in S' by exchanging, at $t = 0$, finitely many numeraire assets, $j \in J$ (with $\# J \leqslant \# \underline{\mathbf{S}}$), which pay off, at $t = 1$, conditionally on the realization of the state. Payoffs are made in quantities of a same commodity bundle, $e \in \mathbb{R}^K \backslash \{0\}$, called the numeraire. We let V^* be the $S \times J$ payoff matrix in numeraire, and V be the corresponding $(S \times L) \times J$ payoff matrix in goods. Non restrictively along De Boisdeffre (2016), we assume that the matrix V^* (or V) and the information structure (S_i) are arbitrage-free. That is, there is no portfolio collection, $(x_i) \in \mathbb{R}^{J \times I}$, such that $\sum_{i \in I} \ z_i = 0$ and $V^*(s) \cdot z_i \geqslant 0$, for every pair $(i, s) \in I \times S_i$, with at least one strict inequality.

For notational puposes, for every $s \in S$, we let $V^*(s) \in \mathbb{R}^J$, be the s^{th} row of matrix V^*, that is, the row of payoffs in numeraire that assets promise to pay if state $s \in S$ prevails. Agents' positions in the different assets are represented by a portfolio, $z \in \mathbb{R}^J$. At the asset price, $q \in \mathbb{R}^J$, and commodity price, $p := (p_s) \in P$, a portfolio, $z \in \mathbb{R}^J$, is thus a contract, which costs $q \cdot z$ units of account at $t = 0$, and promises to pay $(p_s \cdot e) V^*(s) \cdot z$ units of account, in each state $s \in S$, if that state prevails. We denote by $\mathcal{V}$ the set of all $\underline{\mathbf{S}} \times J$ matrices, equiped with the Euclidean norm and topology, and we let $\mathcal{V}^* \subset \mathcal{V}$ be the subset of full rank matrices. For every $p := (p_s) \in P$, we let $V(p) \in \mathcal{V}$ be the matrix whose generic row is $(p_s \cdot e) V^*(s)$, for $s \in \underline{\mathbf{S}}$.

2.2 The consumer's behaviour and concept of equilibrium

Each agent, $i \in I$, receives an endowment, $e_i := (e_{is}) \in \mathbb{R}^{L \times S'_i}_{++}$, granting the commodity bundles, $e_{i0} \in \mathbb{R}^L_{++}$ at $t = 0$, and $e_{is} \in \mathbb{R}^L_{++}$, in each expected state, $s \in S_i$, if it prevails. Given prices, $p := (p_s) \in P$, for commodities, and $q \in \mathbb{R}^J$, for securities, the generic i^{th} agent's consumption set[2] is $X_i := \mathbb{R}_{++}^{L\times S_i'}$ and her budget set is:

$$B_i(p,q) := \{\ (x,z) \in X_i \times \mathbb{R}^J : p_0 \cdot (x_0 - e_{i0}) \leqslant -q \cdot z \ \ and \ \ p_s \cdot (x_s - e_{is}) \leqslant (p_s \cdot e) V^*(s) \cdot z,\ \forall s \in S_i\ \}.$$

Each consumer, $i \in I$, has preferences represented by a utility function, $u_i : X_i \to \mathbb{R}$ and optimises her consumption in the budget set. The above economy is denoted by $\mathcal{E} := \{(I,S,L,J), V, (S_i)_{i\in I}, (e_i)_{i\in I}, (u_i)_{i\in I}\}$ and yields the following equilibrium concept:

Definition 1 *A collection of prices,* $(p,q) \in P \times \mathbb{R}^J$, *and strategies,* $(x_i, z_i) \in B_i(p,q)$, *for each* $i \in I$, *is an equilibrium of the economy,* $\mathcal{E}$, *if the following conditions hold:*

(a) $\forall i \in I,\ (x_i, z_i) \in \arg\max u_i(x)$, *for* $(x,z) \in B_i(p,q)$*;*

(b) $\sum_{i\in I} (x_{is} - e_{is}) = 0,\ \forall s \in \underline{\mathbf{S}}'$*;*

(c) $\sum_{i\in I} z_i = 0$.

The economy, $\mathcal{E}$, is called standard under the following conditions:

Assumption A1 : $\forall i \in I,\ u_i$ *is* C^∞;

Assumption A2 : $\forall i \in I,\ u_i$ *satisfies the Inada Conditions*;

Assumption A3 : $\forall i \in I, \forall x \in X_i,\ Du_i(x) \in X_i$ *(strict monotonicity)*;

Assumption A4: $\forall i \in I,\ h^T D^2 u_i(x) h < 0,\ \forall h \neq 0$;

Assumption A5: $\forall i \in I,\ \forall \overline{x} \in X_i,\ \{\ x \in X_i : u_i(x) \geqslant u_i(\overline{x})\ \}$ *is closed in* X_i.

Assumption A6: *the* $\underline{\mathbf{S}} \times J$ *truncation of* V^* *on* $\underline{\mathbf{S}}$ *has full rank*;

Assumption A7: $\exists z \in \mathbb{R}^J : V^* z \in \mathbb{R}_{++}^S$;

2.3 The model's notations

For convenience, we resume all model's notations hereafter:

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[2] As in Duffie-Shafer (1985), the existence proof relies on interior consumptions. Dropping them could be a next step of research.

- $\mathcal{E} = \{(I, S, L, J), V, (S_i)_{i \in I}, (e_i)_{i \in I}, (u_i)_{i \in I}\}$ summarizes the economy's characteristics. There are two periods, $t \in \{0, 1\}$, finite sets, I, S, L, J, respectively, of consumers, states, goods and assets, a payoff matrix, V (or V^*), information sets, $S_i \subset S$, and $\underline{\mathbf{S}} := \cap_{i \in I} S_i \neq \varnothing$, endowments, e_i, and utilities, u_i, for each $i \in I$.

- We let $s = 0$ be the state at $t = 0$, and denote $\underline{\mathbf{S}}' := \{0\} \cup \underline{\mathbf{S}}$, $S'_i := \{0\} \cup S_i$, and $X_i := \mathbb{R}^{L \times S'_i}_{++}$, the consumption sets, for each $i \in I$.

- $\Delta := \{p \in \mathbb{R}^L_{++} : \|p\| = 1\}$ is the set of anticipated spot prices.

- $P := \mathbb{R}^L_{++} \times \{\ p := (p_s) \in \Delta^S :\ p_s = \overline{p}_s,\ \forall s \in S \backslash \underline{\mathbf{S}}\ \}$, where $(\overline{p}_s) \in \mathbb{R}^{L \times S \backslash \underline{\mathbf{S}}}_{++}$ is given.

- $\mathcal{V}$ is the set of all $\underline{\mathbf{S}} \times J$ matrices and $\mathcal{V}^* \subset \mathcal{V}$ is that of full rank $\underline{\mathbf{S}} \times J$ matrices.

- $V^*(s) \in \mathbb{R}^J$ denotes the row of payoffs in numeraire of V^* in each state $s \in S$. The notation extends to the elements of $\mathcal{V}$.

- For $p \in P$, $V(p) \in \mathcal{V}$ is the matrix whose generic row is $(p_s \cdot e)V^*(s)$, for $s \in \underline{\mathbf{S}}$.

3 The existence Theorem and proof

Our main theorem is as follows:

Theorem 1 *A standard economy, $\mathcal{E}$, with numeraire assets admits an equilibrium.*

The proof builds on standard auxiliary economies, which admit equilibria, $\mathcal{C}^n$, for $n \in \mathbb{N}$, along Theorem 2 of [5]. We proceed as follows: first, we define and set as given auxiliary equilibria. Second, we derive an equilibrium of the original economy, $\mathcal{E}$, from a cluster point of the sequence, $\{\mathcal{C}^n\}_{n \in \mathbb{N}}$, of auxiliary equilibria.

To define auxiliary economies, we introduce an additional fully informed agent, say $i = 1$, having all characteristics of Section 2 (with $S_1 := \underline{\mathbf{S}}$), and we let $I' := I \cup \{1\}$. Then, for every payoff matrix, $V' \in \mathcal{V}$, every collection of prices, $p := (p_s) \in P$ and $q \in \mathbb{R}^J$, endowments, $(e'_i) \in \times_{i \in I'} X_i$, and every agent, $i \in I'$, we define the following budget set:

$$B_i(p, q, V', (e'_i)) := \{ \ (x, z) \in X_i \times \mathbb{R}^J : p_0 \cdot (x_0 - e'_{i0}) \leqslant -q \cdot z,$$

$$p_s \cdot (x_s - e'_{is}) \leqslant [V'(s) + (p_s \cdot e)V^*(s)] \cdot z, \ \forall s \in \underline{\mathbf{S}}$$

$$\textit{and} \ \ p_s \cdot (x_s - e'_{is}) \leqslant (p_s \cdot e)V^*(s) \cdot z, \ \forall s \in S_i \backslash \underline{\mathbf{S}} \ \}.$$

The definition and selection of auxiliary equilibria follow:

Definition 2 *A collection of prices, $(p, q) \in P \times \mathbb{R}^J$, matrix, $V' \in \mathcal{V}$, endowments, $e'_i \in X_i$, and strategies, $(x_i, z_i) \in \ B_i(p, q, V', (e'_i))$, defined for each $i \in I'$, is called an auxiliary equilibrium if the following conditions hold:*

(a) $\forall i \in I', \ (x_i, z_i) \in \arg\max u_i(x)$, *for* $(x, z) \in B_i(p, q, V', (e'_i))$*;*

(b) $\sum_{i \in I'} \ (x_{is} - e'_{is}) = 0, \ \forall s \in \underline{\mathbf{S}}'$*;*

(c) $\sum_{i \in I'} \ z_i = 0$.

Claim 1 *For every $n \in \mathbb{N}$, there exists an auxiliary equilibrium, along Definition 2, $\mathcal{C}^n := (p^n, q^n, V^n, (e^n_i), (x^n_i), (z^n_i))$, such that both following relations hold:*

$\|V^n\| + \|e^n_1\| + \sum_{i \in I} \|e^n_i - e_i\| \leqslant 1/n$ *and* $[V^n + V(p^n)] \in \mathcal{V}^*$.

Moreover, letting $y^n = \sum_{s \in \underline{\mathbf{S}}'} \ p^n_s \cdot e^n_{1s}$, the first agent's equilibrium consumption, x^n_1, is the solution to the problem $x^n_1 \in \arg\max u_1(x)$, for $x \in \{x \in X_1 : \sum_{s \in \underline{\mathbf{S}}'} \ p^n_s \cdot x^n_{1s} = y^n\}$.

Proof Claim 1 is a direct application of Theorem 2 in [5] and proof. □

We now set as given one sequence, $\{\mathcal{C}^n := (p^n, q^n, V^n, (e^n_i), (x^n_i), (z^n_i))\}$, of equilibria, meeting the conditions of Claim 1, and show Theorem 1 from the following Lemma:

Lemma 1 *For the above sequence, $\{\mathcal{C}^n\}$, the following Assertions hold:*

(i) *there exists $\varepsilon > 0$, such that $p_s^n \in [\,\varepsilon, 1\,]^L$, for every $(n, s) \in \mathbb{N} \times \underline{\mathbf{S}}$. Hence, for every $s \in \underline{\mathbf{S}}$, it may be assumed to exist $p_s^* = \lim_{n\to\infty} p_s^n \in [\,\varepsilon, 1\,]^L$;*

(ii) *it may be assumed to exist $(z_i^*) = \lim_{n\to\infty} (z_i^n) \in \mathbb{R}^{J\times I'}$, such that $\sum_{i\in I'} z_i^* = 0$;*

(iii) *it may be assumed to exist $q^* = \lim_{n\to\infty} q^n \in \mathbb{R}^J$;*

(iv) *it may be assumed to exist $p^* = \lim_{n\to\infty} p^n \in P$;*

(v) *it may be assumed to exist $(x_i^*) = \lim_{n\to\infty} (x_i^n)_{i\in I'} \in \mathbb{R}_+^{L\times\underline{\mathbf{S}}'} \times (\times_{i\in I} X_i)$, such that $\sum_{i\in I'} x_{is}^* = \sum_{i\in I} e_{is}$, for every $s \in \underline{\mathbf{S}}'$;*

(vi) *along Assertion (ii)-(v), above, $z_1^* = 0$ and $x_1^* = 0$;*

(vii) *the above collection, $\mathcal{C} := (p^*, q^*, [(x_i^*, z_i^*)]_{i\in I})$, is an equilibrium of the economy $\mathcal{E}$.*

Proof of Lemma 1 See the Appendix. The proof of Theorem 1 is now complete.$\square$

Appendix: Proof of Lemma 1

Assertion (i) Let $s \in \underline{\mathbf{S}}$ and a spot price, $p_s := (p_s^l) \in \mathbb{R}_{++}^L$, in state s be given, such that $\|p_s\| = 1$ (as p_s^n, for every $n \in \mathbb{N}$). The non-negativity and market clearance conditions over auxiliary equilibrium allocations imply that $\{x_{is}^n\}$ is uniformly bounded, for every $i \in I$. From Assumption *A2*, it is, then, standard that, for every $l \in L$, there exists $\alpha > 0$ (independent of $n \in \mathbb{N}$) such that the spot market of good l in state s cannot clear if its market price, p_s^l, satisfies $p_s^l < \alpha$. Assertion (i) follows.$\square$

Assertion (ii) By contraposition, we assume there exists a subsequence, $\{(z_i^{\varphi(n)})\}$, such that $\lim_{n\to\infty} k_{\varphi(n)} := \|(z_i^{\varphi(n)})\| = \infty$, and, to simplify, that $\varphi = Id$. We let

$\alpha := \sup \|e'\| \in \mathbb{R}_{++}$, for $e' \in \{(e'_i) \in \times_{i\in I'} X_i : \|e'_1\| + \sum_{i\in I} \|e'_i - e_i\| \leqslant 1\}$. Then, the definition of auxiliary equilibria yields the following relations:

$$\sum_{i\in I'} z_i^n = 0 \text{ and } [V^n(s) + (p_s^n \cdot e)V^*(s)] \cdot z_i \geqslant -\alpha,\ \forall (i,n,s) \in I' \times \mathbb{N} \times \underline{\mathbf{S}}.$$

For every $(i,n) \in I' \times \mathbb{N}$, we let $z_i'^n := \frac{z_i^n}{k_n}$. The bounded sequence $\{(z_i'^n)_{i\in I'}\}$ admits a cluster point, (z_i), such that $\|(z_i)\| = 1$, and satisfies, from Assertion (i):

$$\sum_{i\in I'} z_i'^n = 0 \text{ and } [V^n(s) + (p_s^n \cdot e)V^*(s)] \cdot z_i'^n \geqslant -\alpha/k_n,\ \forall (i,n,s) \in I' \times \mathbb{N} \times \underline{\mathbf{S}}, \text{ and}$$

$$\sum_{i\in I'} z_i = 0 \text{ and } V(p^*)z_i \geqslant 0,\ \forall i \in I', \text{ when passing to the limit.}$$

From Assumption *A6* and above $V(p^*) \in \mathcal{V}^*$, whereas the above relations imply: $V(p^*)z_i = 0$, for every $i \in I'$. Hence, $(z_i) = 0$, contradicting the above relation, $\|(z_i)\| = 1$. Therefore, the sequence $\{(z_i^n)\}$ is bounded and may be assumed to converge, say to $\{(z_i^*)\}$, and the relations $\sum_{i\in I'} z_i^n = 0$ (for $n \in \mathbb{N}$) yield in the limit: $\sum_{i\in I'} z_i^* = 0$.□

Assertion (iii) Let $n \in \mathbb{N}$ be given. For each $i \in I$, there exist state prices, $\lambda_i^n := (\lambda_{is}^n) \in \mathbb{R}_{++}^{S_i}$, such that $q^n := \sum_{s\in\underline{\mathbf{S}}} \lambda_{is}^n[V^n(s) + (p_s^n \cdot e)V^*(s)] + \sum_{s\in S_i\setminus\underline{\mathbf{S}}} \lambda_{is}^n(p_s^n \cdot e)V^*(s)$ (see Cornet-De Boisdeffre, 2002). It is always possible to normalize the collection of state prices across economies, e.g., to let $\|(\lambda_i^n)\| = 1$, for each $n \in \mathbb{N}$ (see the proof of Theorem 2 in [5]). Doing so bounds the sequence $\{q^n\}$. Assertion (iii) follows. □

Assertion (iv) By the same token as for proving Assertion (i), from Assumptions *A2*-*A7* and Assertions (i)-(iii), there exists an upper bound, $\beta > 0$, beyond which spot markets at $t = 0$ would not clear (given the high the value of endowments, agents would sell and lend cash). So, we may assume that $\{p_0^n\}$ converges to some $p_0^* \in \mathbb{R}_+^L$. Suppose that $p_0^* = 0$. Then again, from Assumptions *A2*-*A7* and Assertions (i)-(iii), for $n \in \mathbb{N}$ big enough, spot markets would not clear at price p_0^n, which contradicts the definition of $\mathcal{C}^n$. Hence, $p_0^* := (p_0^{*l}) \neq 0$. Assume now that $p_0^{*l} = 0$ for some $l \in L$. The same arguments yield the same contradiction and Assertion (iv). □

Assertion (v) From the above Assertions (i)-(ii)-(iii)-(iv), the sequence of allocations, $\{(x_i^n)\}$, is bounded, hence, may be assumed to converge to some $(x_i^*) \in \times_{i'} \mathbb{R}_+^{L \times S_i'}$. Assume, by contraposition, that $x_{is}^{*l} = 0$, for some $(i, s, l) \in I \times S_i' \times L$. Then, Assumption *A2* and the above Assertions would contradict the fact that the sequence, $\{p_s^{nl}\}$, of spot prices, in state s and good l, is bounded. Assertion (v) follows from above and the clearing conditions, $\sum_{i \in I'} (x_i^n - e_i^n)_{s \in \underline{\mathbf{S}}'} = 0$ (for $n \in \mathbb{N}$), passing to the limit. □

Assertion (vi) Claim 1, Assertion (iv) and Assumptions *A2*-*A4* yield: $x_1^* = 0$. Then, it follows from the above Assertions and the budget constaints of the first agent in the auxiliary equilibria, $\mathcal{C}^n$ (for $n \in \mathbb{N}$), which are all binding and pass to the limit, that both relations $-q^* \cdot z_1^* = 0$ and $V(p^*) z_1^* = 0$ hold. From Assumption *A6* and Assertion (i), this implies, $z_1^* = 0$ (since $V(p^*) \in \mathcal{V}^*$). □

Assertion (vii) Let $\mathcal{C} := (p^*, q^*, (x_i^*), (z_i^*))$ be defined from the above Assertions. The collection $\mathcal{C}$ meets conditions (b) and (c) of Definition 1 of equilibrium from the definition. From Assumption *A4* and the definition of auxiliary equilibria, the relations $(x_i^n, z_i^n) \in B_i(p^n, q^n, V^n, (e_i^n))$ and $\{(x_i^n, z_i^n)\} = \arg\max u_i(x)$, for $(x, z) \in B_i(p^n, q^n, V^n, (e_i^n))$, hold, for every $(i, n) \in I \times \mathbb{N}$. Since the closed correspondence, $B_i : (p, q, V', (e_i')) \mapsto B_i(p, q, V', (e_i'))$, and the map, u_i, are continuous, for each $i \in I$, Berge's theorem (see, e.g., Debreu, 1959, p. 19), insures that the latter relations pass to the limit, that is, for every $i \in I$, $(x_i^*, z_i^*) \in B_i(p^*, q^*)$ and $(x_i^*, z_i^*) \in \arg\max u_i(x)$, for $(x, z) \in B_i(p^*, q^*)$. Hence, $\mathcal{C}$ meets condition (a) of Definition 1 and Assertion (vii) holds from above. □

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