# Competitive Equilibrium with Asymmetric Information: an Existence Theorem for Numeraire Assets

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# COMPETITIVE EQUILIBRIUM WITH ASYMMETRIC INFORMATION: AN EXISTENCE THEOREM FOR NUMERAIRE ASSETS

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**Competitive equilibrium with asymmetric information: an existence theorem for numeraire assets**

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## *Abstract*

*In [2], we had extended the classical concepts and arbitrage theory of symmetric information, to an asymmetric information model, which dropped Radner's (1979) rational expectations' assumption. In [3], we showed how agents could infer enough information, in this model, to rule out arbitrage from markets. In [4], we extended to that model Cass' (1984) classical existence Theorem for nominal assets. Namely, we showed that existence of equilibrium was characterized by the generalized no-arbitrage condition introduced in [2], whether agents had symmetric or asymmetric information. We now display the same characteristic property for numeraire asset markets, and, thus, extend Geanakoplos-Polemarchakis' (1986) existence Theorem to the asymmetric information setting. Contrasting with Radner's, these results show that symmetric and asymmetric information economies can be embedded into a common general equilibrium model, where they share similar properties.*

*Key words:* general equilibrium, asymmetric information, arbitrage, existence.

*JEL Classification:* D52.

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# 1 Introduction

When agents are asymmetrically informed, incorporating feedback effects from observing prices or trade volumes into information is essential and, yet, debated. Quoting Ross Starr (1989), "*the theory with asymmetric information is not well understood at all. In short, the exact mechanism by which prices incorporate information is still a mystery and an attendant theory of volume is simply missing.*" A traditional response to that problem is given by the R.E.E. (rational expectations equilibrium) models of asymmetric information, by assuming, quoting Radner (1979), that "*agents have a 'model' or 'expectations' of how equilibrium prices are determined*". Along this assumption, agents may infer private information of other agents from comparing actual prices and price expectations with theoretical values at a price revealing equilibrium. This presumes much of agents' forecast and inference capacacities and leads to standard cases of inexistence of equilibrium.

Our approach does not use Radner's assumption. In [2], we drop rational expectations and provide the basic tools, concepts and properties for an arbitrage theory, embedding jointly (as particular application cases) the symmetric and asymmetric information settings, into a same model. In [4], we prove that a financial equilibrium with nominal assets exists in this model, not only generically - as with rational expectations - but under the same no-arbitrage condition, with symmetric or asymmetric information, namely, under the general no-arbitrage condition introduced in [2], which characterizes the existence of equilibrium. This result extends to asymmetric information Cass' (1984) classical existence Theorem for nominal assets.

With differential information and no price model a la Radner, the question arises why, and how, the above characteristic no-arbitrage condition should hold. To that aim, we show in [3] that agents with no price model may always infer, in finitely many steps, enough information, from observing trade opportunities on financial markets, to preclude arbitrage. Whence reached, this information cannot be refined, equilibrium always exists and equilibrium prices, whatever they be, reveal no more information.

We now introduce a similar model, with numeraire assets, replacing nominal securities, and display the same property, namely, that the no-arbitrage condition of [2] characterizes the existence of financial equilibrium, in that model. This result extends the classical Theorem of symmetric information for numeraire assets, due to Geanakoplos-Polemarchakis (1986). The above outcomes show that dropping the rational expectation hypothesis to deal with asymmetric information is, not only possible, but also improves the existence properties of the model. Agents, with no price model, may then infer, from observing markets, the information they require. Moreover, full existence is restored, replacing Radner's (1979) generic one. Then, symmetric and asymmetric information are only two sides, or application cases, of the same broad general equilibrium model.

Formally, the model we present is a two-period pure exchange economy, where agents, possibly asymmetrically informed, face uncertainty, at the first period, on which state of nature will randomly prevail tomorrow, out of a finite state space. Agents exchange consumption goods on spot markets, and securities on financial markets, which pay off in numeraire (i.e., in a given commodity or commodity bundle), so as to transfer wealth across periods and states. Asymmetric information may stem from agents' possible private knowledge that some states cannot prevail.

The paper is organized as follows: we present the model and existence Theorem, in Section 2, and the Theorem's proof, in Section 3. An Appendix proves Lemmas.

# 2 The basic model

In this Section, we recall the framework and results of the model introduced in [2], replacing nominal by numeraire assets. We present a pure-exchange economy with two periods ($t \in \{0, 1\}$), a commodity market and a financial market. At $t = 0$, agents are uncertain which state of nature will prevail at $t = 1$. All sets, of agents, $I := \{1, ..., m\}$, goods, $\mathcal{L} := \{1, ..., L\}$, states, $S$, assets, $\mathcal{J} := \{1, ..., J\}$, are finite.

## 2.1 The model's notations

Throughout, we denote by $\cdot$ the scalar product and $\|.\|$ the Euclidean norm and let $s = 0$ be the non-random state at $t = 0$ and $S' := \{0\} \cup S$. We let $2^K$ be the set of non-empty subsets of a given set, $K$. For all elements, $\Sigma \in 2^{S'}$ and $(\Sigma_i)_{i \in I} \in (2^S)^m$, and tuples, $(\varepsilon, s, l, x, x', y, y') \in ]0, 1] \times \Sigma \times L \times \mathbb{R}^{\Sigma} \times \mathbb{R}^{\Sigma} \times \mathbb{R}^{L\Sigma} \times \mathbb{R}^{L\Sigma}$, we denote by:

- $x_s \in \mathbb{R}$, $y_s \in \mathbb{R}^L$ the scalar and vector, indexed by $s \in \Sigma$, of $x$, $y$, respectively;
- $y_s^l$ the $l^{th}$ component of $y_s \in \mathbb{R}^L$;
- $x \leqslant x'$ and $y \leqslant y'$ (respectively, $x << x'$ and $y << y'$) the relations $x_s \leqslant x'_s$ and $y_s^l \leqslant y_s'^l$ (resp., $x_s < x'_s$ and $y_s^l < y_s'^l$) for each $(l, s) \in \{1, ..., L\} \times \Sigma$;
- $x < x'$ (resp., $y < y'$) the joint relations $x \leqslant x'$, $x \neq x'$ (resp., $y \leqslant y'$, $y \neq y'$);
- $\mathbb{R}_+^{L\Sigma} = \{x \in \mathbb{R}^{L\Sigma} : x \geqslant 0\}$ and $\mathbb{R}_+^{\Sigma} := \{x \in \mathbb{R}^{\Sigma} : x \geqslant 0\}$,
  $\mathbb{R}_{++}^{L\Sigma} := \{x \in \mathbb{R}^{L\Sigma} : x >> 0\}$ and $\mathbb{R}_{++}^{\Sigma} := \{x \in \mathbb{R}^{\Sigma} : x >> 0\}$,
- $\Delta_0 := \{(p_0, q) \in \mathbb{R}_+^L \times \mathbb{R}^J : \|p_0\| + \|q\| = 1\}$ and $\Delta := \{p \in \mathbb{R}_+^L, \ \|p\| = 1\}$;
- $\Sigma_i^c := \Sigma_i \setminus \cap_{j=1}^m \Sigma_j := \{s \in \Sigma_i : s \notin \cap_{j=1}^m \Sigma_j\}$, for each $i \in I$.

### 2.2 The commodity and asset markets

Agents exchange the $L$ consumption goods at both periods on spot markets to increase their welfare. Trade may occur because the generic agent, $i \in I$, has an endowment, $e_i := (e_{is}) \in \mathbb{R}^{LS'}_{++}$, which grants the commodity bundles, $e_{i0} \in \mathbb{R}^L_{++}$, at $t = 0$, and, $e_{is} \in \mathbb{R}^L_{++}$, in each $s \in S$, if this state prevails. Ex post, her welfare is measured by a continuous utility index, $u_i : \mathbb{R}^{2L}_+ \to \mathbb{R}_+$, over consumptions at both dates.

The financial market permits limited transfers across periods and states, via $J$ assets, or securities, $j \in \mathcal{J} := \{1, ..., J\}$, exchanged at $t = 0$ and paying off at $t = 1$, in numeraire, that is, in a fixed commodity (bundle), $e \in \mathbb{R}^L_+$, which we take such that $\|e\| = 1$. For each $j \in \mathcal{J}$, we let $v^j := (v^j_s) \in \mathbb{R}^S$ be the $j^{th}$ asset's flow of random payoffs (in numeraire) across states. This defines $V := (v^j)$ as the $(\#S \times J)$ payoff matrix, which is henceforth set as given and always referred to. For each $s \in S$, we denote by $V(s) \in \mathbb{R}^J$ the $s^{th}$ row vector of matrix $V$. Redundant assets are eliminated $(J = rankV)$ and the financial market may also be incomplete $(J < \#S)$.

Agents may take unrestrained positions (positive, if purchased; negative, if sold), in each security, which defines their portfolios. At market prices, $q \in \mathbb{R}^J$, on financial markets, and $p \in \mathbb{R}^{LS}_+$, on spot markets, a portfolio, $z \in \mathbb{R}^J$, is a contract, which costs $q \cdot z$ units of account at $t = 0$, and whose cash payoff in each state, $s \in S$, will be $(p_s \cdot e)V(s) \cdot z$, if state $s$ prevails. We henceforth normalize prices, that is, we restrict first period prices to the set $\Delta_0$, and spot prices in each state to $\Delta$.

### 2.3 Information signals and refinements

As in [2], each agent, $i \in I$, receives a private information signal, $S_i \subset S$, at $t = 0$, informing her that the true state will be in $S_i$. Henceforth, the collection, $(S_i)$, of all signals is set as given and we let $\underline{\mathbf{S}} := \cap_{i=1}^m S_i$. Agents are correctly informed, in the sense that no state of $S \backslash \underline{\mathbf{S}}$ can prevail. They refine their beliefs along Definition 1:

**Definition 1** *A collection, $(\Sigma_i)$, of $m$ subsets of $S$ is called an information structure, and we denote it by $(\Sigma_i) \in \mathcal{IS}$, if their intersection is non-empty, that is, if:*

*(a) $\underline{\mathbf{\Sigma}} := \cap_{i=1}^m \Sigma_i \neq \varnothing$.*

*Let $(\Sigma_i) \in \mathcal{IS}$ be given. A collection, $(\pi_i)$, of probabilities on $S$, such that the relation $\Sigma_i = \{s \in S : \pi_i(s) > 0\}$ holds for each $i \in I$, is said to support $(\Sigma_i)$, and called a structure of beliefs. An information structure $(\Sigma'_i) \in \mathcal{IS}$ is said to refine $(\Sigma_i)$, and we denote it $(\Sigma'_i) \leq (\Sigma_i)$, if it meets the following relations:*

*(b) $\Sigma'_i \subset \Sigma_i$, $\forall i \in I$.*

*A structure, $(\Sigma'_i) \in \mathcal{IS}$, is said to be a self-attainable refinement of $(\Sigma_i)$ if:*

*(c) $(\Sigma'_i) \leq (\Sigma_i)$ and $\cap_{i=1}^m \Sigma'_i = \cap_{i=1}^m \Sigma_i$.*

With no loss of generality, we henceforth restrict structures and refinements, $(\Sigma_i) \in \mathcal{IS}$, to be consistent with agents' prior information signals, that is, $(\Sigma_i) \leq (S_i)$.

### 2.4 Consumers' behavior and the notion of equilibrium

At $t = 0$, agents are assumed to make decisions after having reached a (final self-attainable) refinement, $(\Sigma_i) \leq (S_i)$ and a supporting structure of beliefs, $(\pi_i)$. All agents, $i \in I$, observe the market prices, $\omega_0 := (p_0, q) \in \Delta_0$, at $t = 0$, forecast identical spot prices, $p \in \Delta^{\underline{\mathbf{S}}}$, in all realizable states, and fictitious idiosyncratic prices, $p_i \in \mathbb{R}_{++}^{L(\Sigma_i \backslash \underline{\mathbf{S}})}$ (whenever $\Sigma_i \neq \underline{\mathbf{S}}$). For the sake of homogeneous notations, but costlessly, we will also let $\{p_i\} = \varnothing$ and, thus, refer to the 'anticipation $p_i$' for every $i \in I$, such that $\Sigma_i = \underline{\mathbf{S}}$. The generic $i^{th}$ agent consumption set, budget set and utility function are defined as follows, respectively:

$X(\Sigma_i) := \mathbb{R}_+^{L\Sigma'_i}$ and

$$B_i(\Sigma_i,\omega_0,p,p_i) := \{\ (x,z)\in X(\Sigma_i)\times\mathbb{R}^J:\ p_0\cdot(x_0-e_{i0})\leqslant -q\cdot z$$
$$\textit{and}\quad p_s\cdot(x_s-e_{is})\leqslant(p_s\cdot e)V(s)\cdot z,\ \forall s\in\underline{\mathbf{S}}$$
$$\textit{and}\ \ p_{is}\cdot(x_s-e_{is})\leqslant(p_{is}\cdot e)V(s)\cdot z,\ \forall s\in\ \Sigma_i\backslash\underline{\mathbf{S}}\ \}$$

and the V.N.M. utility, $u_i^{\pi_i}: x\in X(\Sigma_i)\ \mapsto \sum_{s\in\Sigma_i}\pi_i(s)\ u_i(x_0,x_s)$.

The generic $i^{th}$ agent elects a strategy, which maximises her utility function in the buget set, i.e., a strategy of the set $B_i^*(\Sigma_i,\omega_0,p,p_i) := \arg\max_{(x,z)\in B_i(\Sigma_i,\omega_0,p,p_i)}\ u_i^{\pi_i}(x)$. This economy is denoted by $\mathcal{E}$. Its financial equilibrium is defined as follows:

**Definition 2** *A collection of market prices, $\omega_0 := (p_0,q)\in\Delta_0$, at $t=0$, and $p\in\Delta^{\underline{\mathbf{S}}}$, at $t=1$, a self-attainable refinement, $(\Sigma_i)\leq(S_i)$, its supporting beliefs, $(\pi_i)$, idiosyncratic anticipations, $(p_i)$, and strategies, $(x_i,z_i)\in B_i(\Sigma_i,\omega_0,p,p_i)$, defined as above, for each $i\in I$, is an equilibrium of the economy $\mathcal{E}$, if:*

*(a)* $\forall i\in I,\ (x_i,z_i)\in B_i^*(\Sigma_i,\omega_0,p,p_i) := \arg\max_{(x,z)\in B_i(\Sigma_i,\omega_0,p,p_i)}\ u_i^{\pi_i}(x)$;

*(b)* $\forall s\in\underline{\mathbf{S}}',\ \sum_{i=1}^m(x_{is}-e_{is})=0$;

*(c)* $\sum_{i=1}^m z_i=0$.

*Under the above conditions, the prices, $\omega_0$, $p$ and $(p_i)$, the refinement, $(\Sigma_i)$, or the structure of beliefs, $(\pi_i)$, are said to (jointly) support the equilibrium.*

### 2.5 No-arbitrage prices and the information they reveal

We first recall from [2] the definition of arbitrage-free prices and structures.

**Definition 3** *Let a structure, $(\Sigma_i)\in\mathcal{IS}$, and a price, $q\in\mathbb{R}^J$, be given. The information structure, $(\Sigma_i)$, is said to to be $q$-arbitrage-free (hence, arbitrage-free), or $q$ to be a common no-arbitrage price of $(\Sigma_i)$, if the following equivalent Conditions hold:*

*(a) there is no agent, $i\in I$, and portfolio $z_i\in\mathbb{R}^J$, such that $-q\cdot z_i\geqslant 0$ and $V(s)\cdot z_i\geqslant 0$ for every $s\in\Sigma_i$, with at least one strict inequality;*

$(b)$ *for every* $i \in I$*, there exists* $\lambda_i \in \mathbb{R}^{\Sigma_i}_{++}$*, such that* $q = \sum_{s \in \Sigma_i} \lambda_{is} V(s)$*.*

*We let* $Q_c[(\Sigma_i)]$ *be the set of common no-arbitrage prices of* $(\Sigma_i)$*. The structure* $(\Sigma_i)$ *is said to be arbitrage-free (resp.,* $q$*-arbitrage-free) if* $Q_c[(\Sigma_i)] \neq \varnothing$ *(resp., if* $q \in Q_c[(\Sigma_i)]$*). We say that* $q$ *is a no-arbitrage price (resp., a self-attainable no-arbitrage price) of* $(\Sigma_i)$ *if there exists a refinement (resp., a self-attainable refinement),* $(\Sigma_i^*) \leq (\Sigma_i)$*, such that* $q \in Q_c[(\Sigma_i^*)]$*, and we denote their set by* $Q[(\Sigma_i)]$*, which is non-empty.*

We now summarize the main results of [2] into the following Claim.

**Claim 1** *Let* $(\Sigma_i) \leq (S_i) \in \mathcal{IS}$ *and* $q \in \mathbb{R}^J$*, be given. The following assertions hold:*

$(i)$ the structure $(\Sigma_i)$ *is arbitrage-free if and only if there exists no portfolio collection,* $(z_i) \in \mathbb{R}^{Jm}$*, such that* $\sum_{i=1}^m z_i = 0$ *and* $V(s_i) \cdot z_i \geqslant 0$*, for every pair* $(i, s_i) \in I \times \Sigma_i$*, with at least one strict inequality;*

$(ii)$ *there exists a coarsest arbitrage-free refinement of* $(S_i)$*, denoted by* $(\overline{\mathbf{S}}_i)$*, which is self-attainable;*[2]

$(iii)$ *there exists a coarsest* $q$*-arbitrage-free refinement of* $(S_i)$*, denoted by* $(\mathbf{S}_i(q))$ *and said to be revealed by price* $q$*, if and only if* $q \in Q[(S_i)]$*, i.e.,* $q$ *is a no-arbitrage price;*

$(iv)$ $(\mathbf{S}_i(q)) \leq (S_i)$ *is self-attainable if and only if* $q \in Q[(S_i)]$ *is self-attainable.*

**Proof** see Cornet-De Boisdeffre (2002). □

We say the economy, $\mathcal{E}$, is standard if it meets the following Conditions:

***Assumption A1,*** $\forall i \in I,\ e_i >> 0$;

***Assumption A2,*** $\forall i \in I$, $u_i$ *is class* $C^1$*, strictly increasing, strictly quasi-concave.*

We now state the existence Theorem.

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[2] *coarsest in that every arbitrage-free refinement of* $(S_i)$ *is a refinement of* $(\overline{\mathbf{S}}_i)$*. Hence,* $(\overline{\mathbf{S}}_i)$ *is unique.*

**Theorem 1** *Let a standard economy, $\mathcal{E}$, the structure, $(S_i)$, a self-attainable refinement $(\Sigma_i) \leq (S_i)$, supported by beliefs, $(\pi_i)$, and anticipations, $(p_i)$, be given, defined as above. Then, the following Assertions hold:*

*$(i)$ if $(\pi_i)$ and $q \in \mathbb{R}^J$ jointly support a financial equilibrium, then, $q \in Q_c[(\Sigma_i)]$;*

*$(ii)$ if $(\Sigma_i)$ is arbitrage-free, then, $(\pi_i)$ & $(p_i)$ jointly support a financial equilibrium.*

We prove Assertion $(i)$, hereafter, and Assertion $(ii)$, in Section 3.

**Proof of Assertion** $(i)$ Let a structure of beliefs, $(\pi_i)$, support a refinement, $(\Sigma_i) \leq (S_i)$ and, jointly with a price, $q \in \mathbb{R}^J$, support a financial equilibrium. We assume, by contraposition that $q \notin Q_c[(\Sigma_i)]$. Then, from Condition $(a)$ of Definition 3, there exist $i \in I$, a state $s_i \in \Sigma_i$, and a portfolio $z \in \mathbb{R}^J$, such that $-q \cdot z \geqslant 0$, $(V(s_i) \cdot z - q \cdot z) > 0$ and $V(s) \cdot z \geqslant 0$ for every $s \in \Sigma_i$. We let $p \in \Delta^{\mathbf{S}}$, and $\overline{p}_i \in \mathbb{R}^{L(\Sigma_i \setminus \mathbf{S})}$ and $(x_i, z_i) \in B_i(\Sigma_i, \omega_0, p, \overline{p}_i)$, be, respectively, the equilibrium price, and the $i^{th}$ agent's equilibrium anticipation and strategy. Denoting $\overline{p} := (p, \overline{p}_i) \in \mathbb{R}^{L\Sigma'_i}$, we let the reader check, as standard from Condition $(a)$ of Definition 2 and Assumption *A2*, that $\overline{p} >> 0$ and, from the above inequalities, that there exists $x'_i \in X(\Sigma_i)$, such that $(x'_i, z_i + z) \in B_i(\Sigma_i, \omega_0, p, \overline{p}_i)$ and $x'_i > x_i$ $\big($hence, $u_i^{\pi_i}(x'_i) > u_i^{\pi_i}(x_i)\big)$. This contradicts the fact that the equilibrium strategy, $(x_i, z_i)$, meets Condition $(a)$ of Definition 2. □

# 3 The Theorem's proof

Throughout, we set as given a standard economy, $\mathcal{E}$, the information structure, $(S_i)$, a self-attainable refinement, $(\Sigma_i) \leq (S_i)$, supporting beliefs, $(\pi_i)$, and anticipations, $(p_i)$, defined as above. From above, we need only prove Assertion $(ii)$ of Theorem 1, so, we henceforth assume that $(\Sigma_i) \leq (S_i)$ is arbitrage-free. The proof's principle is to construct a sequence of purely financial economies, which all admit an equilibrium from Theorem 1 of [4]. An equilibrium in each auxiliary economy is, hence, set as given. Then, from the sequence of auxiliary equilibria, we derive an equilibrium of the initial economy, $\mathcal{E}$.

### 3.1 Auxiliary economies, $\mathcal{E}^n$, and equilibria, $\mathcal{C}^n$

We set as given $p^0 \in \Delta^{\underline{\mathbf{S}}} \cap \mathbb{R}^{L\underline{\mathbf{S}}}_{++}$ (as denoted in sub-Section 2.1) Then, we define by induction, for each $n \in \mathbb{N}\backslash\{0\}$, a sequence, $\{p^n\}$, of prices in $\Delta^{\underline{\mathbf{S}}} \cap \mathbb{R}^{L\underline{\mathbf{S}}}_{++}$, which are equilibrium prices, at $t = 1$, of economies, $\mathcal{E}^n$, defined hereafter.

Let $n \in \mathbb{N}\backslash\{0\}$ be given and assume that $p^{n-1} \in \Delta^{\underline{\mathbf{S}}} \cap \mathbb{R}^{L\underline{\mathbf{S}}}_{++}$ has been defined at the previous step of induction. The auxiliary economy, $\mathcal{E}^n$, is of the type described in [4]. It has the same periods, $t \in \{0, 1\}$, goods, $l \in \mathcal{L}$, agents, $i \in I$, number of assets, $j \in \mathcal{J}$, information signals, $(\Omega_i)$, anticipations, $(p_i)$, as above. Referring to the notations and definitions of the economy $\mathcal{E}$, its characteristics are as follows:

- The information structure, $(\Omega_i)$, is a collection of sets, $\Omega_i := \underline{\mathbf{S}} \cup \widetilde{S}_i$, defined for each $i \in I$. The set $\widetilde{S}_i$ is either empty (if $\Sigma_i = \underline{\mathbf{S}}$) or defined as a subset of $I \times S$, namely, $\widetilde{S}_i := \{i\} \times \Sigma_i^c := \{i\} \times \Sigma_i \backslash \underline{\mathbf{S}}$ (if $\Sigma_i \neq \underline{\mathbf{S}}$). It then consists of purely formal states, $\widetilde{s}_i$, none of which will prevail at $t = 1$. Contrarily, $\underline{\mathbf{S}} = \cap_{i=1}^m \Omega_i$ is the set of realizable states of the economy $\mathcal{E}^n$. We let $s = 0$ be the non-random state at $t = 0$, $\Omega_i' := \{0\} \cup \Omega_i$, for each $i \in I$, and $\Omega := \cup_{i=1}^m \Omega_i$ be the state space of $\mathcal{E}^n$.
- In each un-realizable state, $\widetilde{s}_i := (i, s) \in \widetilde{S}_i$, the generic $i^{th}$ agent has exactly one sure anticipation of the spot price to prevail, namely $p_{is} \in \mathbb{R}^L_{++}$, given above.
- In each realizable state, $s \in \underline{\mathbf{S}}$, the generic $i^{th}$ agent has perfect foresight, i.e., anticipates with certainty the true equilibrium price, denoted by $p^n_s \in \mathbb{R}^L$.
- The generic $i^{th}$ agent's endowment, $E_i := (E_{is}) \in \mathbb{R}^{\Omega_i'}_{++}$, in the economy $\mathcal{E}^n$, is defined with reference to the endowment, $e_i$, in the economy $\mathcal{E}$. Namely, $E_{is} := e_{is}$, for each state $s \in \underline{\mathbf{S}}$, and $E_{i\widetilde{s}_i} := e_{is}$, for each fictitious state $\widetilde{s}_i := (i, s) \in \widetilde{S}_i$.

- The $\Omega \times J$ payoff matrix, $V^n := (V^n(s))_{s\in\Omega}$, of the economy $\mathcal{E}^n$, is defined by $V^n(s) := (p_s^{n-1} \cdot e)V(s)$, for each $s \in \underline{\mathbf{S}}$, and $V^n(\widetilde{s}_i) := (p_{is} \cdot e)V(s)$, for each $i \in I$ and each $\widetilde{s}_i := (i,s) \in \widetilde{S}_i$ (if $\Sigma_i \neq \underline{\mathbf{S}}$). The payoff matrix is, hence, **purely nominal**.

- For every pair of market prices, $\omega_0^n := (p_0^n, q^n) \in \Delta_0$ and $p^n \in \Delta^{\underline{\mathbf{S}}}$ the generic $i^{th}$ agent has for consumption set, budget set and utility function, respectively:

$X_i := \mathbb{R}_+^{L\Omega_i'}$,

identified to $X(\Sigma_i)$ via the one-to-one mapping, $\varphi_i : x \in X_i \mapsto x^* := \varphi_i(x) \in X(\Sigma_i)$, defined by $x_s^* := x_s$, for each $s \in \underline{\mathbf{S}}$, and $x_s^* := x_{\widetilde{s}_i}$, for each $\widetilde{s}_i := (i,s) \in \widetilde{S}_i := \{i\} \times \Sigma_i^c$;

$$\begin{aligned} B_i^n(\Omega_i, \omega_0^n, p^n, p_i) \quad := \quad \{\ & (x,z) \in X_i \times \mathbb{R}^J : p_0^n \cdot (x_0 - e_{i0}) \leqslant -q^n \cdot z \\ & \textit{and} \quad p_s^n \cdot (x_s - e_{is}) \leqslant V^n(s) \cdot z, \ \forall s \in \underline{\mathbf{S}} \\ & \textit{and} \quad p_{is} \cdot (x_{\widetilde{s}_i} - e_{is}) \leqslant V^n(\widetilde{s}_i) \cdot z, \ \forall \widetilde{s}_i := (i,s) \in \widetilde{S}_i\ \}; \end{aligned}$$

$U_i := u_i^{\pi_i} \circ \varphi_i$, along the above identification mapping, $\varphi_i$.

A Corollary of Theorem 1 in [4] yields Lemma 1.

**Lemma 1** *The generic economy, $\mathcal{E}^n$ (for $n \in \mathbb{N}\backslash\{0\}$), admits an equilibrium, namely, a collection of prices, $\omega_0^n := (p_0^n, q^n) \in \Delta_0$ and $p^n \in \Delta^{\underline{\mathbf{S}}}$, and strategies, $(x_i^n, z_i^n) \in B_i^n(\Omega_i, \omega_0^n, p^n, p_i)$, defined for each $i \in I$, such that:*

*(i)* $\forall i \in I,\ (x_i^n, z_i^n) \in \arg\max_{(x,z)\in B_i^n(\Omega_i, \omega_0^n, p^n, p_i)}\ U_i(x)$;

*(ii)* $\forall s \in \underline{\mathbf{S}}',\ \sum_{i=1}^m\ (x_{is}^n -\ e_{is}) = 0$;

*(iii)* $\sum_{i=1}^m z_i^n = 0$.

*Moreover, the equilibrium prices, $p^n \in \Delta^{\underline{\mathbf{S}}}$, and allocation, $(x_i^n)$, are such that:*

*(iv)* $x_{is}^n \in [0,\alpha]^L$, $\forall (n,i,s) \in \mathbb{N}\backslash\{0\} \times I \times \underline{\mathbf{S}}'$, *with* $\alpha := \max_{(s,l)\in S'\times\mathcal{L}} \sum_{i=1}^m e_{is}^l > 0$;

*(v)* $\exists \varepsilon > 0 : p_s^{nl} \geqslant \varepsilon$, $\forall (n,s,l) \in \mathbb{N}\backslash\{0\} \times \underline{\mathbf{S}} \times \mathcal{L}$.

**Proof** see the Appendix. □

From Lemma 1, for all $n > 0$, we set as given an equilibrium of the economy $\mathcal{E}^n$:

$$\mathcal{C}^n := \{\ \omega_0^n \in \Delta_0,\ p^n \in \Delta^{\underline{\mathbf{S}}}, (x_i^n, z_i^n) \in B_i^n(\Omega_i, \omega_0^n, p^n, p_i),\ for\ each\ i \in I\}$$

always referred to, which meets the Conditions of Lemma 1. The equilibrium price, $p^n \in \Delta^{\underline{\mathbf{S}}} \cap \mathbb{R}_{++}^{L\underline{\mathbf{S}}}$, permits to pursue the induction and define the $(n+1)$-economy, $\mathcal{E}^{n+1}$, in the same way as above, and auxiliary economies, $\mathcal{E}^{n'}$, for every $n' \in \mathbb{N}\backslash\{0\}$.

We state a Lemma, which serves to prove Theorem 1.

**Lemma 2** *The following assertions hold:*

$(i)$ *it may be assumed to exist* $(\omega_0^*{=}(p_0^*, q^*), p^*) := \lim_{n\to\infty}(\omega_0^n, p^n) \in \Delta_0 \times \Delta^{\underline{\mathbf{S}}}$*;*

$(ii)$ *it may be assumed to exist* $(z_i^*) = \lim_{n\to\infty}(z_i^n)_{i\in I} \in \mathbb{R}^{Jm}$*, such that* $\sum_{i=1}^m z_i^* = 0$*;*

$(iii)$ *it may be assumed to exist* $(x_i) = \lim_{n\to\infty}(x_i^n)_{i\in I}$*, such that* $\sum_{i=1}^m (x_{is}-e_{is})_{s\in\underline{\mathbf{S}}'} = 0$*.*

*With the above identification mapping,* $\varphi_i$*, we let* $x_i^* := \varphi_i(x_i) \in X(\Sigma_i)$*, for each* $i \in I$*.*

**Proof** see the Appendix. □

### 3.2 An equilibrium of the economy $\mathcal{E}$

We now prove Theorem 1, via Claim 2.

**Claim 2** *The collection of prices,* $(\omega_0^*, p^*) \in \Delta_0 \times \Delta^{\underline{\mathbf{S}}}$*, refinement,* $(\Sigma_i) \leq (S_i)$*, beliefs,* $(\pi_i)$*, anticipations,* $(p_i)$*, allocation,* $(x_i^*)$*, portfolios,* $(z_i^*) = \lim_{n\to\infty}(z_i^n)$*, defined from Lemma 2, is an equilibrium of the economy* $\mathcal{E}$*.*

**Proof** Let $\mathcal{C}^* = \{\ (\omega_0^*,\ p^*),\ (\pi_i),\ (x_i^*, z_i^*)\ for\ each\ i \in I\ \}$ be defined as in Claim 2. From Lemma 2, $\mathcal{C}^*$ meets Conditions $(b)$-$(c)$ of Definition 2 of equilibrium. We show that $\mathcal{C}^*$ also satisfies the relation $(x_i^*, z_i^*) \in B_i(\Sigma_i, \omega_0^*, p^*, p_i)$, for every $i \in I$, besides Condition $(a)$ of Definition 2.

Let $i \in I$ be given. From the definition of $\mathcal{C}^n$, for all $n \in \mathbb{N}\backslash\{0\}$, the relations $p_0^n \cdot (x_{i0}^n - e_{i0}) \leqslant -q^n \cdot z_i^n$ and $p_s^n \cdot (x_{is}^n - e_{is}) \leqslant (p_s^{n-1} \cdot e) V(s) \cdot z_i^n$ and $p_{is'} \cdot (x_{i\widetilde{s}_i}^n - e_{is'}) \leqslant (p_{is'} \cdot e) V(s') \cdot z_i^n$ hold, for each $s \in \underline{\mathbf{S}}$ and each $\widetilde{s}_i = (i, s') \in \widetilde{S}_i$, and yield, in the limit, from Lemma 2 and the continuity of the scalar product, $p_0^* \cdot (x_{i0}^* - e_{i0}) \leqslant -q^* \cdot z_i^*$ and $p_s^* \cdot (x_{is}^* - e_{is}) \leqslant (p_s^* \cdot e) V(s) \cdot z_i^*$, for each $s \in \underline{\mathbf{S}}$, and $p_{is'} \cdot (x_{is'}^* - e_{is'}) \leqslant (p_{is'} \cdot e) V(s') \cdot z_i^*$, for each $s' \in \Sigma_i^c$, that is, $(x_i^*, z_i^*) \in B_i(\Sigma_i, \omega_0^*, p^*, p_i)$.

Assume, by contraposition, that $\mathcal{C}^*$ fails to meet Condition $(a)$ of Definition 2, then, there exist $i \in I$, $(x, z) \in B_i(\Sigma_i, \omega_0^*, p^*, p_i)$ and $\varepsilon \in \mathbb{R}_{++}$, such that:

$$(I) \qquad \varepsilon + u_i^{\pi_i}(x_i^*) < u_i^{\pi_i}(x).$$

We may assume that there exists $\delta \in \mathbb{R}_{++}$, such that:

$$(II) \qquad x_s^l \geqslant \delta, \text{ for every } (s, l) \in \Sigma_i' \times \mathcal{L}.$$

If not, for every $\alpha \in [0, 1]$, we define the strategy $(x^\alpha, z^\alpha) := ((1-\alpha)x + \alpha e_i, (1-\alpha)z)$, which belongs to $B_i(\Sigma_i, \omega_0^*, p^*, p_i)$, a convex set. From Assumption *A1*, the strategy $(x^\alpha, z^\alpha)$ meets relations $(II)$ whenever $\alpha > 0$. Moreover, from relation $(I)$ and the continuity of $u_i^{\pi_i}$, the strategy $(x^\alpha, z^\alpha)$ also meets relation $(I)$, for every $\alpha > 0$, small enough. So, we may indeed assume relations $(II)$.

We let the reader check, as immediate from the relations $(I)$–$(II)$, $(x, z) \in B_i(\Sigma_i, \omega_0^*, p^*, p_i)$ and $(\omega_0^*, p^*) \in \Delta_0 \times \Delta^{\underline{\mathbf{S}}} \cap \mathbb{R}_{++}^{L\underline{\mathbf{S}}}$ (which holds from Lemma 1), from Assumption *A2*, and continuity arguments, that we may also assume there exists $\gamma \in \mathbb{R}_{++}$, such that:

$$(III) \qquad \begin{cases} p_0^* \cdot (x_0 - e_{i0}) \leqslant -\gamma - q^* \cdot z \\ p_s^* \cdot (x_s - e_{is}) \leqslant -\gamma + (p_s^* \cdot e) V(s) \cdot z, \ \forall s \in \underline{\mathbf{S}} \\ p_{is} \cdot (x_s - e_{is}) \leqslant -\gamma + (p_{is} \cdot e) V(s) \cdot z \ , \ \forall s \in \Sigma_i \backslash \underline{\mathbf{S}} \end{cases}$$

From $(III)$, Lemma 2-$(i)$, scalar product continuity, there exists $N \in \mathbb{N}$, such that:

$$(IV) \qquad \left\{ \begin{array}{l} p_0^n \cdot (x_0 - e_{i0}) \leqslant -q^n \cdot z \\ p_s^n \cdot (x_s - e_{is}) \leqslant V^n(s) \cdot z, \ \forall s \in \underline{\mathbf{S}} \\ p_{is} \cdot (x_s - e_{is}) \leqslant V^n(\widetilde{s}_i) \cdot z, \ \forall \widetilde{s}_i := (i, s) \in \widetilde{S}_i \end{array} \right\}, \text{ for every } n \geqslant N.$$

Let $\varphi_i : X_i \to X(\Sigma_i)$ be the identification mapping defined above Lemma 1 and $\overline{x} := \varphi_i^{-1}(x) \in X_i$ be given. From $(IV)$ and the definitions, the relations $U_i(\overline{x}) = u_i^{\pi_i}(x)$ and $(\overline{x}, z) \in B_i^n(\Omega_i, \omega_0^n, p^n, p_i)$ hold, for each $n \geqslant N$. Then, from equilibrium conditions on $\mathcal{C}^n$, Lemma 2-$(iii)$ and the continuity of $u_i^{\pi_i}$, there exists $n \geqslant N$, such that:

$$(V) \qquad U_i(\overline{x}) := u_i^{\pi_i}(x) \leqslant U_i(x_i^n) := u_i^{\pi_i}(\varphi_i(x_i^n)) < \varepsilon + u_i^{\pi_i}(x_i^*).$$

The above relations $(I)$ and $(V)$ yield, jointly: $u_i^{\pi_i}(x) < \varepsilon + u_i^{\pi_i}(x_i^*) < u_i^{\pi_i}(x)$. This contradiction proves that $\mathcal{C}^*$ meets Condition $(a)$ of Definition 2, and, from above, is an equilibrium of the economy $\mathcal{E}$. The proof of Theorem 1 is now complete. □

# Appendix: proof of the Lemmas

**Lemma 1** *The generic economy, $\mathcal{E}^n$ (for $n \in \mathbb{N} \backslash \{0\}$), admits an equilibrium, namely, a collection of prices, $(\omega_0^n := (p_0^n, q^n), p^n) \in \Delta_0 \times \Delta^{\underline{\mathbf{S}}}$, and strategies, $(x_i^n, z_i^n) \in B_i^n(\Omega_i, \omega_0^n, p^n, p_i)$, defined for each $i \in I$, such that:*

$(i)$ $\forall i \in I, \ (x_i^n, z_i^n) \in \arg\max_{(x,z) \in B_i^n(\Omega_i, \omega_0^n, p^n, p_i)} \ U_i(x)$;

$(ii)$ $\forall s \in \underline{\mathbf{S}}', \ \sum_{i=1}^m (x_{is}^n - \ e_{is}) = 0$;

$(iii)$ $\sum_{i=1}^m z_i^n = 0$.

*Moreover, the equilibrium prices, $p^n \in \Delta^{\underline{\mathbf{S}}}$, and allocation, $(x_i^n)$, are such that:*

$(iv)$ $x_{is}^n \in [0, \alpha]^L$, $\forall (n, i, s) \in \mathbb{N} \backslash \{0\} \times I \times \underline{\mathbf{S}}'$, *with* $\alpha := \max_{(s,l) \in S' \times \mathcal{L}} \sum_{i=1}^m e_{is}^l > 0$;

$(v)$ $\exists \varepsilon > 0 : p_s^{nl} \geqslant \varepsilon$, $\forall (n, s, l) \in \mathbb{N} \backslash \{0\} \times \underline{\mathbf{S}} \times \mathcal{L}$.

**Proof** Let $n \in \mathbb{N}\backslash\{0\}$ be given throughout. The economy $\mathcal{E}^n$ is, formally, one of the type presented in [4]. From the induction argument in sub-Section 3.1, $p^{n-1} \in \Delta^{\underline{\mathbf{S}}} \cap \mathbb{R}^{L\underline{\mathbf{S}}}_{++}$, which implies that $p^{n-1}_s \cdot e > 0$, for each $s \in \underline{\mathbf{S}}$. Then, from Definition 3, the fact that $(\Sigma_i)$ is arbitrage-free (relative to the payoff matrix $V$) implies that $(\Omega_i)$ is arbitrage-free on a purely financial market (relative to the payoff matrix $V^n$). Therefore, along Theorem 1 of [4] and its proof, the economy $\mathcal{E}^n$ admits an equilibrium, that is, a pair of prices, $\omega^n_0 = (p^n_0, q^n) \in \Delta_0$ and $p^n \in \Delta^{\underline{\mathbf{S}}}$ and strategies, $(x^n_i, z^n_i) \in B^n_i(\Omega_i, \omega^n_0, p^n, p_i)$, for each $i \in I$, which satisfy Conditions $(i)$-$(ii)$-$(iii)$ of Lemma 1. Moreover, the relations $(x^n_{is})_{s\in\underline{\mathbf{S}}'} \geqslant 0$ and $\sum_{i=1}^m (x^n_{is} - e_{is})_{s\in\underline{\mathbf{S}}'} = 0$ hold from Lemma 1-$(ii)$ and imply that $x^n_{is} \in [0,\alpha]^L$, for each $(i,s) \in I\times\underline{\mathbf{S}}'$, with $\alpha := \max_{(s,l)\in S'\times\mathcal{L}} \sum_{i=1}^m e^l_{is} > 0$.

Let $\beta := \sup_{i\in I,(x,y)\in[0,\alpha]^{2L},(l,l')\in\mathcal{L}^2} \frac{\partial u_i}{\partial y^{l'}}(x,y)/\frac{\partial u_i}{\partial y^l}(x,y)$. From Lemma 1-$(i)$ and Assumption *A2*, it is standard that $\beta \in \mathbb{R}_+$ and that $p^n >> 0$. We show that $\frac{p^{nl}_s}{p^{nl'}_s} \leqslant \beta$, for every $(s,(l,l')) \in \underline{\mathbf{S}} \times \mathcal{L}^2$. Otherwise, it is immediate (from Lemma 1-$(i)$ and Assumptions *A1*-*A2*) that there exist $i \in I$, $(s,(l,l')) \in \underline{\mathbf{S}} \times \mathcal{L}^2$ and $x \in X_i$, identical to $x^n_i$ in every component but two, $x^{l'}_s := x^{nl'}_{is} + \frac{\delta}{p^{nl'}_s}$ and $x^l_s := x^{nl}_{is} - \frac{\delta}{p^l_s}$ (for $\delta \in \mathbb{R}_{++}$ small enough), such that $(x, z^n_i) \in B^n_i(\Omega_i, \omega^n_0, p^n, p_i)$ and $U_i(x) > U_i(x^n_i)$, in contradiction with the fact that the equilibrium strategy, $(x^n_i, z^n_i)$, meets Condition $(i)$ of Lemma 1. We let the reader check that the joint relations $p^n_s >> 0$, $\|p^n_s\| = 1$ and $\frac{p^{nl}_s}{p^{nl'}_s} \leqslant \beta$, which hold, from above, for each $(s,(l,l')) \in \underline{\mathbf{S}}\times\mathcal{L}^2$, imply $p^{nl}_s \geqslant \varepsilon := \frac{1}{\beta\sqrt{L}}$, for each $(s,l) \in \underline{\mathbf{S}} \times \mathcal{L}$. □

**Lemma 2** *The following assertions hold:*

$(i)$ *it may be assumed to exist* $(\omega^*_0{=}(p^*_0, q^*), p^*) := \lim_{n\to\infty}(\omega^n_0, p^n) \in \Delta_0 \times \Delta^{\underline{\mathbf{S}}}$*;*

$(ii)$ *it may be assumed to exist* $(z^*_i) = \lim_{n\to\infty}(z^n_i)_{i\in I} \in \mathbb{R}^{Jm}$*, such that* $\sum_{i=1}^m z^*_i = 0$*;*

$(iii)$ *it may be assumed to exist* $(x_i) = \lim_{n\to\infty}(x^n_i)_{i\in I}$*, such that* $\sum_{i=1}^m (x_{is}{-}e_{is})_{s\in\underline{\mathbf{S}}'} = 0$.

**Proof** Assertion $(i)$ The proof is immediate from the compactness of $\Delta_0 \times \Delta^{\underline{\mathbf{S}}}$. □

Assertion $(ii)$ For each $i \in I$, we let $Z_i^0 := \{z \in \mathbb{R}^J : V(s) \cdot z = 0, \forall s \in \Sigma_i\}$ be a vector space, $Z_i^{\perp}$ be its orthogonal complement in $\mathbb{R}^J$ and let $Z^0 = \sum_{i=1}^m Z_i^0$ and $\mathcal{Z} := \{(z_i) \in \times_{i \in I} Z_i^{\perp} : \sum_{i=1}^m z_i \in Z^0\}$. For each $n \in \mathbb{N}\backslash\{0\}$, we denote by $(z_i^n) = (z_i^{no}) + (z_i^{n\perp})$ the orthogonal decomposition of $(z_i^n)$ on $\times_{i \in I}(Z_i^0 \times Z_i^{\perp})$, and we show that the sequence $\{(z_i^{n\perp})\}_{n \in \mathbb{N}^*}$ is bounded.

Indeed, we let $\varepsilon \in ]0, \inf_{(i,s) \in I \times \Sigma_i^c} p_{is}^l[$ satisfy condition $(v)$ of Lemma 1 and define the positive number $\delta := \max_{i \in I} \frac{\|e_i\|(1+\|p_i\|)}{\varepsilon}$. The budget constraints and market clearance conditions on the equilibrium, $\mathcal{C}^n$, imply, from Lemma 1 and above:

$(I)$ $\qquad [(z_i^{n\perp}) \in \mathcal{Z}$ *and* $V(s_i) \cdot z_i^{n\perp} \geqslant -\delta$, $\forall (i, s_i) \in I \times \Sigma_i]$, for every $n \in \mathbb{N}^*$.

Assume, by contradiction, that $\{(z_i^{n\perp})\}$, is unbounded, i.e., there exists an extracted sub-sequence $\{(z_i^{\varphi(n)\perp})\}$, such that $n < \|(z_i^{\varphi(n)\perp})\| \leqslant n+1$, for all $n \in \mathbb{N}^*$. From $(I)$, the portfolios $(z_i^{*n}) := \frac{1}{n}(z_i^{\varphi(n)\perp})$ satisfy $1 < \|(z_i^{*n})\| \leqslant 1 + \frac{1}{n}$, for all $n \in \mathbb{N}^*$, and:

$(II)$ $\qquad [(z_i^{*n}) \in \mathcal{Z}$ *and* $V(s_i) \cdot z_i^{*n} \geqslant -\frac{\delta}{n}$, $\forall (i, s_i) \in I \times \Sigma_i]$, for every $n \in \mathbb{N}^*$.

From $(II)$, the continuity of the scalar product and above, the sequence $\{(z_i^{*n})\}$ may be assumed to converge, say to $(\overline{z}_i) \in \mathcal{Z}$, a closed set, such that $\|(\overline{z}_i)\|$=1 and:

$(III)$ $\qquad [(\overline{z}_i) \in \mathcal{Z}$ *and* $V(s_i) \cdot \overline{z}_i \geqslant 0$, $\forall (i, s_i) \in I \times \Sigma_i]$.

Relations $(III)$, the fact that $(\Sigma_i)$ is arbitrage-free (relative to $V$), Claim 1 and above yield $(\overline{z}_i) \in \times_{i \in I} Z_i^0 \cap \mathcal{Z} = \{0\}$, which contradicts the above relation, $\|(\overline{z}_i)\| = 1$. Hence, the sequence, $\{(z_i^{n\perp})\}$, is in $\mathcal{Z}$, and is bounded. The definition of $\mathcal{Z}$ yields a bounded sequence, $\{(\tilde{z}_i^{no})\}$, such that $(\tilde{z}_i^{no}) \in \times_{i \in I} Z_i^0$ and $\sum_{i=1}^m (\tilde{z}_i^{no} + z_i^{n\perp}) = 0$, for every $n \in \mathbb{N}\backslash\{0\}$. For each $n \in \mathbb{N}\backslash\{0\}$, we may obviously replace $(z_i^{no})$ by $(\tilde{z}_i^{no})$ in the definition of equilibrium portfolios. Hence, we may assume that $\{(z_i^{no})\} = \{(\tilde{z}_i^{no})\}$. Then, the sequence, $\{(z_i^n)\} = \{(z_i^{no}) + (z_i^{n\perp})\}$ is bounded from above, and may be assumed to converge, say to $(z_i^*) := \lim_{n\to\infty} (z_i^n)_{i\in I} \in \mathbb{R}^{Jm}$ and the relations $\sum_{i=1}^m z_i^n = 0$, which hold, from Lemma 1, for every $n \in \mathbb{N}\backslash\{0\}$, pass to the limit and yield: $\sum_{i=1}^m z_i^* = 0$. $\square$

Assertion $(iii)$ From the equilibrium relations in the auxiliary economies and compactness arguments, we need only prove that the sequence $\{(x_i^n)\}_{n\in\mathbb{N}^*}$ is bounded. Let $P := 1+\max_{i\in I}\|p_i\|$, $\beta = \max_{i\in I}\|e_i\|$, $W := \max_{s\in S}\|V(s)\|$, $Z := \sup_{n\in\mathbb{N}^*}\|(z_i^n)\|$, along Lemma 2-$(ii)$ be given. Let $\alpha := \max_{(s,l)\in S'\times\mathcal{L}} \sum_{i=1}^m e_{is}^l$, and $\varepsilon > 0$ be given bounds along Lemma 1. We may take $\varepsilon < p_{is}^l$ for every triple $(i,l,s) \in I\times\mathcal{L}\times\Sigma_i^c$. Then, for each $(n,i,\widetilde{s}_i{=}(i,s)) \in \mathbb{N}\backslash\{0\}\times I\times\widetilde{S}_i$, Assertions $(i)$ & $(iv)$ of Lemma 1 yield, in steps: $p_{is}\cdot(x_{i\widetilde{s}_i}^n - e_{is}) = \sum_{l=1}^L p_{is}^l\cdot(x_{i\widetilde{s}_i}^{nl} - e_{is}^l) \leqslant PWZ$, hence, $p_{is}^l\cdot x_{i\widetilde{s}_i}^{nl} \leqslant PWZ + P\beta$, for each $l\in\mathcal{L}$; then, $\|x_{i\widetilde{s}_i}^n\| \leqslant \gamma := LP[WZ+\beta]/\varepsilon$ and, finally, $\|(x_i^n)\| \leqslant m(\#S{+}1)(\gamma+\alpha)$. $\square$


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