Heterogeneous Immigration, Segregation and Trade

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Abstract: By introducing taste heterogeneity between mobile workers in a New Economic Geography (NEG) model where the housing price is the main driving force behind dispersion of workers we show that residential segregation and agglomeration are not the sole stable equilibria and that dispersion also emerges with trade liberalization. In addition we find that contrary to the Tiebout hypothesis where segregation is efficient, here it is the dispersed and mixed equilibrium which can be improving for all.

JEL Classifications: F12, R12

Keywords: Segregation, Heterogeneous preferences, Economic geography

1. Introduction

Needless to say, urban segregation is an important issue. Economic costs of gated communities on one side and of poor districts on the other are high. Among direct costs, the fact that productive forces remain inactive on account of a bad zip code is certainly not tolerable. Among indirect costs, the underground economy which is often fueled by spatial and social exclusion represents a significant waste of resources that should be avoided. Then it urges us to understand the determinant of spatial segregation.

There are at least three theories which aim to comprehend segregation. The first one focuses on land rent: the price of land is higher around places where services and jobs are concentrated owing to low commuting costs, then individuals are spread according to their incomes, the rich are located near attractive places while the poor are compelled to live far away. In other words, the land use pattern forms a set of Thünen rings (Fujita [6]). This theory explains one part of segregation, for instance in many European countries - from the second world war to the seventies - massive constructions were undertaken at the periphery of cities because vast areas were available at a low price. A second theory considers that the source of segregation comes from discrimination: homogeneous community might either be willing to pay to avoid immigrants which do not share their characteristics - this phenomenon is known as decentralized discrimination - or may be able to restrict immigrants' location choices; in that case sociologists speak about centralized discrimination. The latter kind of discrimination was particularly common in the US at the beginning of the twentieth century; in Chicago for instance, the *Hyde Park Improvement Protective Club* produced different listings of housings for sale according to racial factors. Nowadays and since the sixties, a symptom of decentralized discrimination has been revealed by the "white flight" that takes place when the concentration of black families reaches a certain threshold (the famous tipping point of Schelling [11]). Lastly a third theory asserts that a part of the segregation comes from preference heterogeneity. Taste heterogeneity may concern the consumption of public goods and/or the consumption of private goods. In the case where individuals do not share identical tastes concerning public goods, those who value these goods will concentrate in locations where public services are abundant, while those with a lower demand will choose locations with a lower level of public services and taxes. To sum up, individuals reveal their preferences by "voting with their feet" (Tiebout [12]). In such a framework, competition among jurisdictions results in homogeneous clubs, no individual will be better off moving, and then segregation is efficient. Obviously, as has been shown empirically by Rhode and Strumpf [10], non-Tiebout motives also matter, individuals can choose segregation deliberately because it is a way to get a job thanks to the community networks or to consume typical goods which are less available in other communities. The usual example to illustrate this kind of segregation concerns the Chinese community (Chinatowns in USA), but many others examples exist, for instance in Canada a Francophile resident may prefer Montreal to Toronto because a wide variety of goods that suit his/her tastes are available there (Glazer et al. [7]). In a recent work, Zeng [13] gives the first theoretical formalization of this kind of segregation. By using an NEG standard model (the Footloose Entrepreneurs model), the author finds an important result: full dispersion, in which two types of mobile workers and two industries equally disperse in two regions, is not a stable equilibrium. In other words, only ghettos can exist and a situation where people are melt throughout territory is not possible.

In the present work we propose to challenge this point of view by again raising the question of the existence of a dispersed equilibrium and by questioning the desirability of segregation in a modified version of the Zeng model. Intuitively in the Zeng model a new immigrant increases demand on the goods market and thus encourages firms to move, but if a firm follows mobile workers, then firms' competition leads to a reduction in the local price index, which encourages immigration but deters delocation. Then even in the case where firms are dispersed, consumers are not and segregation emerges as the sole stable equilibrium. But Zeng puts aside the land market although the availability of land is often a major element that impacts on segregation. Then in the present model, by introducing an immobile housing stock, we find that segregation, i.e. agglomeration of each type of workers in a different location pushes up housing prices and then makes the dispersive equilibrium stable. Although this centrifugal force has been studied before,[1] its introduction in a framework with heterogeneous preferences provides new insights. In particular, as simple as this model may be, it provides interesting equilibrium outcomes: for intermediate level of trade freeness agglomeration and segregation are stable while for lower and for higher trade costs the stable multiple equilibria are dispersion and segregation. This result once again reveals the importance of History, but also indicates that the trend of segregation which has increased rapidly in the latest decades can be broken by an exogenous shock or by public policies. Significantly while Zeng finds that segregation is the best outcome for all immigrants, we find that the equilibrium where workers are dispersed and mingled can be Pareto dominant.

All these results are presented in four stages: in Section 2 the formal model is developed, Section 3 is devoted to the equilibrium analysis, Section 4 investigates welfare and lastly Section 5 concludes.

2. The model

The economic space is made of two regions, 1 and 2. The economy has four sectors, two modern sectors (X, Y), a traditional sector (T) and a housing sector (H). The two modern sectors produce a continuum of varieties of a horizontally differentiated product under increasing returns. Each variety is produced by a single firm, using both skilled and unskilled labor. The T-sector produces homogeneous good - which are trade costless - under constant returns, exclusively using unskilled workers, which are equally spread in the two regions.

[1] See Candau [1] for a survey.

Each household's preferences take the Dixit [4] form introduced by Pflüger [7] in the NEG literature. We extend this framework to the case of two kinds of skilled workers and two industries (*à la* Zeng [13]) and we follow Pflüger and Suedekum [4] by including the housing consumption:

$$U_i^x = \eta\mu \ln X_i^x + (1-\eta)\mu \ln Y_i^x + \theta \ln H_i^x + T_i^x$$
$$U_i^y = (1-\eta)\mu \ln X_i^y + \eta\mu \ln Y_i^y + \theta \ln H_i^y + T_i^y$$

Where X_i^k and Y_i^k are two kinds of manufactured goods consumed by a representative consumer of type $k \in \{x, y\}$ in a city $i \in \{1,2\}$, H_i^k denotes the housing consumption, T_i^k is the homogeneous commodity. The term θ represents the preference for housing, while μ and η express the intensity of preferences for the differentiated products with the following assumption concerning η:

$$0.5 < \eta < 1$$

which characterizes the fact that k-type individuals prefer type-k industrial goods to any other type.

X_i^k and Y_i^k are represented by:

$$X_i^k = \left[\int_{Nx} d_i^{kx}(s)^{\zeta} ds\right]^{\zeta}, Y_i^k = \left[\int_{Ny} d_i^{ky}(s)^{\zeta} ds\right]^{\zeta}$$

With $\zeta = (\sigma-1)/\sigma$ and N^x the number of varieties produced by the industry x, σ the elasticity of substitution between varieties, and d_i^{kx} the consumption of a manufactured product x by a consumer of type $k \in \{x, y\}$.

We denote s_{h^x} the population share of entrepreneurs x in region 1, then with h the total population of each entrepreneur's type and with n_j^k the set of all varieties in industry $k \in \{x, y\}$ produced in region $j \in \{1,2\}$ we have:

$$n_1^x + n_1^y = (s_{h^x} + s_{h^y})h, \quad n_1^x + n_1^y = ((1-s_{h^x}) + (1-s_{h^y}))h$$
$$n_1^x + n_1^y = N^x, \quad n_1^x + n_1^y = N^y$$

With $p_{ji}^k(s)$ the price of a variety consumed in region i, the demand $d_{ji}^{xk}(s)$ for variety consumed by type-x workers in location i is given by:

$$d_{ji}^{xx}(s) = \mu\eta p_{ji}^x(s)^{-\rho}/(P_i^x)^{1-\rho}, \quad d_{ji}^{xy}(s) = \mu(1-\eta)p_{ji}^y(s)^{-\rho}/(P_i^y)^{1-\rho}$$

and the demand $d_{ji}^{yk}(s)$ consumed by type-y workers of in location i is:

$$d_{ji}^{yx}(s) = \mu(1-\eta)p_{ji}^x(s)^{-\rho}/(P_i^x)^{1-\rho}, \quad d_{ji}^{yy}(s) = \mu\eta p_{ji}^y(s)^{-\rho}/(P_i^y)^{1-\rho}$$

where prices indices are given by:

$$P_i^x = \left[\int_{s\in n_i^x} p_{ii}^x(s)^{1-\rho} ds + \int_{s\in n_j^x} p_{ji}^x(s)^{1-\rho} ds\right]^{1/(1-\rho)},$$
$$P_i^y = \left[\int_{s\in n_i^y} p_{ii}^y(s)^{1-\rho} ds + \int_{s\in n_j^y} p_{ji}^y(s)^{1-\rho} ds\right]^{1/(1-\rho)}.$$

The demand for housing is given by $H_i = \theta / p_i^H$ then the aggregate housing demand in region 1 is $H_1 = \theta((s_{h^x} + s_{h^y})h + \frac{L}{2})/p_1^H$. Obviously at the equilibrium aggregate demand equals aggregate supply, which is fixed and given by H in either region. Hence the housing price is calculated in region 1 as follows:

$$p_i^H = \theta((s_{h^x} + s_{h^y}) + L/2)h/\overline{H}$$

According to this equation, housing price increases with the number of inhabitants.

Concerning the cost function in the industrial sector, we consider with Forslid and Ottaviano[5] that the fixed cost and the marginal cost are associated with different factors: the fixed cost involves f units of entrepreneurs while the variable cost requires v units of unskilled workers. Furthermore, because skilled workers are totally mobile from one sector to the next, and have the same skills (there is no productivity difference between x and y) there is equalization of wages.

Thus the total cost of producing q(s) units of a typical manufactured variety is:

$$TC_i = fw_i + vw^T q(s)$$

Where w^T represents the wage of unskilled workers. Each firm is a monopolist on the production of its variety, then when a typical type-k firm in city 1 maximizes its profit under the Dixit-Stiglitz assumptions, it sets the following price on its local market:

$$p_{ii}(s) = vw^T \rho / (\rho - 1).$$

Because there is free entry, profits are always equal to zero, which gives the level of output

$$q_i(s) = (\rho - 1) fw_i / vw^T.$$

Moreover industrial varieties are exchanged between regions under trade costs which take the form of iceberg costs: if an industrial variety produced in market i is sold at price p_{ii} there, then the delivered price (c.i.f) of that variety in market j is going to be $\tau p_{ij}(s)$ with $\tau > 1$. Then we obtain a change in price indices:

$$P_i^x = (n_i^x + \phi n_j^x)^{1/(1-\rho)} / \mathrm{C}, \quad P_i^y = C(n_i^y + \phi n_j^y)^{1/(1-\rho)} / C$$

with $C = (\rho - 1) / vw^T \rho$ a constant and with $\phi = \tau^{1-\sigma}$ an indicator of trade opening ($\phi \in [0,1]$, with 0 for autarky and 1 for free trade)

Total demand in region 1 becomes equal to:

$$x_1 = (s_{h^x} h + L/2) d_{11}^{xx} + (s_{h^y} h + L/2) d_{11}^{yx} + ((1 - s_{h^x}) h + L/2) \tau d_{12}^{xx} + ((1 - s_{h^y}) h + L/2) \tau d_{12}^{yx},$$

$$with: d_{11}^{xx}(s) = C\mu\eta / n_1^x + \phi n_2^x, \quad d_{11}^{yx}(s) = C\mu(1-\eta) / n_1^x + \phi n_2^x$$

$$\tau d_{12}^{xx} = C\mu\eta\phi / (\phi n_1^x + n_2^x), \quad \tau d_{12}^{yx} = C\mu(1-\eta)\phi / \phi n_1^x + n_2^x$$

By considering the numerator of this demand x_1, we can observe that from the dispersed equilibrium an increase in the number of entrepreneurs in region 1, increases expenditure there and lowers it abroad, which, ceteris paribus and as long as there are impediments to trade, increases demand in region 1 (market access effect). Concerning the denominator an increase in the number of firms in region 1 fosters a decrease in the demand addressed to a typical firm in this region (local competition effect).

3. Equilibrium Analysis

In the long run migration stops when real wages are equalized in case of symmetry, or when agglomeration in one city generates a higher relative real wage. More precisely it is assumed that migration is regulated by a simple Marshallian adjustment[2]:

$$\dot{s}_{h^x} = \gamma^x (V_1^{h^x} - V_2^{h^x}), \quad \dot{s}_{h^y} = \gamma^y (V_1^{h^y} - V_2^{h^y})$$

Where γ^x and γ^y are the adjustment speeds and $V_i^{h^k}$ the indirect utility of a type-k entrepreneur in region i, defined by:

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[2]This is an usual assumption in the literature.

$$V_i^{h^x} = Y_i^{h^x} - \mu\eta \ln P_i^x - \mu(1-\eta)\ln P_i^y - \theta \ln p_{Hi}$$
$$V_i^{h^y} = Y_i^{h^y} - \mu\eta \ln P_i^y - \mu(1-\eta)\ln P_i^x - \theta \ln p_{Hi}$$

Notice that in the long run two additional forces appear: on the one hand the term $\theta \ln p_{Hi}$ introduces a dispersive force, namely the land market crowding effect, which implies that an increase in the number of skilled workers in one city impacts negatively on the cost of living there. On the other hand the term $\mu\eta \ln P_i^x - \mu(1-\eta)\ln P_i^y$ indicates that goods are cheaper in a central place because imports are lower and thus the burden of transaction costs too. Then agglomeration also has a positive effect on the cost of living.

With this model in hand we want to know which of the following equilibria exist and are stable:

1) Full segregation of migrants (type-x individuals in one region and type-y individuals in the other).
2) Total agglomeration (each type agglomerated in one location).
3) Full dispersion (50% of individuals x and y in region 1).

In the following we analyze these three different cases successively[3].

3.1 Full segregation

Full segregation is assumed in this subsection. We consider that individuals x are all agglomerated in region 1, while type-y individuals are agglomerated in region 2. Then we set $s_{hx} = 1$, $s_{hy} = 0$, which gives $n_1^y = (\lambda^x + \lambda^y)h - n_1^x = h - n_1^x$, $n_2^y = n_1^x + (2 - \lambda^x - \lambda^y)h - N^x = n_1^x + h - N^x$, and $n_2^x = N^x - n_1^x$. Next, by equalizing the demand to the supply, we obtain the following system:

$$q_1 = x_1 = y_1, \;\; q_2 = x_2 = y_2$$

Which allows us to obtain the expression of w_1, n_1^x, and N^x:

$$w_1 = w_2 = (h+L)\mu / h\rho$$
$$n_1^x = h(L(1-\phi) + 2h\eta(1+\phi) - 2h\phi)/[2(h+L)(1-\phi)]$$
$$N^x = N^y = h$$

Then an increase in the heterogeneity of preferences η, induces an increase in the process of specialization (or in other words in the agglomeration of industries).

Therefore, it is important to notice that in this model full segregation of workers is not equivalent to full specialization; indeed because all individuals consume all goods (x and y) the geographical separation of individuals does not necessarily imply a full separation of industries. Two industries are partially dispersed if $n_1^x \in]H/2, H[$, which according to the above expression of n_1^x is equivalent to:

$$\phi < \phi^{FS} \equiv \left(2h(1-\eta) + L\right) / \left(L + 2h\eta\right)$$

This partial separation is stable if there is no incentive to migrate, i.e if $V_1^{h^x} > V_2^{h^x}$, which is verified here:

$$V_1^{h^x} - V_2^{h^x} = \left(\mu(1+2\eta)\ln \tfrac{L+2h\eta}{2h+L-2h\eta}\right) / \left(\rho - 1\right) > 0$$

[3]As in Zeng ([4], Appendix B) the case of partial segregation (for instance individuals of type-x in one region and individuals of type-y dispersed in the two regions) is not stable.

Equivalently it is found that $V_1^{h^y} - V_2^{h^y} < 0$, then whatever the type of individuals, segregating workers is a stable equilibrium[4].

We can now analyze the equilibrium with a full separation of industries. In notation, $\lambda^x = 1$, $\lambda^y = 0$, and $n_1^x = n_2^y = N^x = N^y = H$ hold, which gives the following nominal wages:

$$w_1 = w_2 = (h+L)\mu / h\rho$$

This full segregation is stable if mobile workers of type-x located in region 1 earn a higher real wage in this location, i.e if $V_1^{h^x} > V_2^{h^x}$ is verified, which is always true, indeed we find that:

$$V_1^{h^x} - V_2^{h^x} = \mu((1-2\eta)\ln\phi)/(\rho-1)$$

which is positive because by definition we have $\ln\phi < 0$ (because $\phi < 1$) and $(1-2\eta) < 0$.

Symmetrically we find that $V_1^{h^y} < V_2^{h^y}$ always holds.

We can now turn toward firms' behavior in order to determine if they can make positive profit by shifting their production from region 1 to region 2. Profit in region 2 is given by:

$$[\nu x_2/(\rho-1)] - w_2 = \mu(1-\phi)\left[L(\phi-1) + 2h(\eta(1+\phi)-1)\right]/2h\rho\phi,$$

this expression is negative, and then separation is stable if:

$$\phi > \phi^{FS} \equiv (2h(1-\eta)+L)/(L+2h\eta)$$

Then the following result concludes our analysis by:

Proposition 1: *A separation of workers and a partial dispersion of firms is stable if trade liberalization is below a critical value* ϕ^{FS}*, while a full separation is stable above this threshold.*

Such a result is very close to the proposition 2 presented by Zeng [13]. Furthermore we can notice that full separation emerges sooner (in terms of trade freeness) for high preference heterogeneity. A corollary of this is that for homogeneous preference the full separation of industries never happens (indeed full separation occurs if $\phi > (2h+L)/L > 1$, which is impossible by definition).

We can now investigate the agglomerated equilibrium which turns out to be more complex.

3.2 Total agglomeration in one region

Let's assume that all the activities are agglomerated in region 1, in that case we set $\lambda^x = \lambda^y = 1$, and $n_1^x = n_2^y = N^x = N^y = N$ and we find the following nominal wages:

$$w_1 = (h+2L)\mu/\rho$$

$$w_2 = \mu\left[L + (h+L)\phi^2\right]/\rho\phi(2h-1)$$

Full agglomeration is stable if $V_1^{h^x} - V_2^{h^x} > 0$ which is verified when:

$$N/(2h-1)(\rho-1)\rho\phi < 0$$

with $N = \theta\rho\left[\rho-1-2h(\rho-1)\right]\phi\ln\left[(2+L)/L\right] + \mu((1-\rho)(h\phi(1-2h+\phi)+L(1+(2-4h)\phi+\phi^2)) + (2h-1)$.

Even if it is not possible to find the range of trade liberalization for which the full agglomeration is stable, we can present the following result:

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[4]We have also checked that no firms find an interest in moving when $\phi < (2h+L-2h\eta)/(L+2h\eta)$, which is verified by:

$$\partial x_1/\partial n_1^x < 0,\quad \partial y_1/\partial n_1^y < 0,\quad \partial x_2/\partial n_1^x > 0,\quad \partial y_2/\partial n_2^y > 0$$

Proposition 2: *Full agglomeration is a stable equilibrium for intermediate value of trade freeness.*

Proof: *Figure 1 helps to understand this proof, the vertical axis represents the differential real wages while the horizontal axis displays the level of trade freeness*[5]. *Then it is clear in this simulation that the welfare gap between region 1 and 2 is bell-shaped with respect to* ϕ *and positive for intermediate values of trade freeness. In order to prove this result we need to verify that the term* $V_1^{h^x} - V_2^{h^x}$ *is 1) negative around extreme values of* ϕ*, 2) increasing around* $\phi = 0$ *and increasing around* $\phi = 1$*, 3) admits only one maximum. Firstly we can show that* $V_1^{h^x} - V_2^{h^x}|_{\phi=0} < 0$ *because* $\lim_{\phi\to 0} N = -L\mu(\rho-1) < 0$ *and* $\lim_{\phi\to 0}(2h-1)(\rho-1)\rho\phi = 0^+$, *then* $\lim_{\phi\to 0}[V_1^{h^x} - V_2^{h^x}] = -\infty$. *Concerning the derivative we get:*

$$\partial(V_1^{h^x} - V_2^{h^x})/\partial\phi = \mu(\phi(\rho - 2h\rho + h\phi - h\phi\rho) - L(\rho-1)(\phi^2-1)/[(2h-1)(\rho-1)\rho\phi^2]$$

and then we find that the limit of the numerator when ϕ *tends to zero is positive (equals to* $L\mu(\rho-1)$*) while the denominator tends to 0 then* $\frac{\partial(V_1^{h^x} - V_2^{h^x})}{\partial\phi}|_{\phi=0} > 0$ *. Around* $\phi = 1$ *and by normalizing the total population h to one (without loss of generality), we find that* $V_1^{h^x} - V_2^{h^x} = -\theta\ln((2+L)/L) < 0$ *and* $\partial(V_1^{h^x} - V_2^{h^x})/\partial\phi|_{\phi=1} = \mu(1-2\rho)/[(\rho-1)\rho] < 0$ *. Lastly the real wage gap admits only one maximum between 0 and 1 given by* $\phi^{\max} = [-\rho + \sqrt{4L(1+L)(\rho-1)^2 + \rho^2}\,]/2(1+L)(\rho-1)$ *.* ***QED***

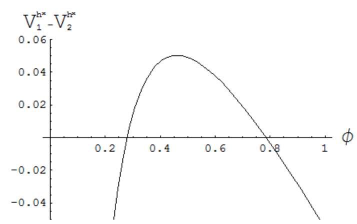

Figure 1. Welfare gap

These intermediate values for which agglomeration is sustainable are noted ϕ_l^s and ϕ_h^s (in Figure 1 we get approximately $\phi_l^s = 0.28$ and $\phi_h^s = 0.78$).

3.3 Full dispersion

Full dispersion is characterized with

$$n_2^x = N^x - n_1^x,\ n_1^y = (\lambda^x + \lambda^y)h - n_1^x,\ n_2^y = n_1^x + (2 - \lambda^x - \lambda^y)h - N^x$$

This allows us to obtain at the symmetric equilibrium the expression of w_1, n_1^x and N^x:

$$w_1 = w_2 = (h+L)\mu/h\rho$$
$$n_1^x = [-h\mu - L\mu + 2hw_1\rho]/2w_1\rho = h/2$$
$$N^x = h$$

which permits to deduce the following proposition:

[5]Parameters: $\mu = 0.6$, $\rho = 5$, $L = h = 1$, $\theta = 0.05$.

Proposition 3: *A stable interior equilibrium, in which two types of mobile workers disperse, exists for an intermediate level of trade liberalization.*

Proof: *see appendix A. **QED**.*

This result contrasts with Zeng [13] who proves the absence of this equilibrium.

3.4 Evolving path

To summarize the previous findings, Figure 2.a and 2.b illustrate the distribution of firms and mobile workers as a function of trade freeness. In order to describe these Figures, it is important to recall that four effects determine location choices: the 'market-access effect', the 'cost-of-living effect', the 'local competition effect', and the 'land market crowding effect'. While the market access effect entails a growth in nominal wages when workers move to the North, the cost-of-living effect results in a price reduction which fosters workers' agglomeration and deters that of firms. Limiting the effects of these agglomeration forces on the workers' location choice, the competition effect and the land market crowding effect, play an opposite role and represent the two dispersion forces of the model.

Around autarky, dispersive forces are dominant for firms which are partially dispersed while agglomeration forces lead to a segregation of workers. Trade liberalization in such a case leads to a gradual specialization of region 1 and 2 until the threshold ϕ^{FS} where the full separation equilibrium emerges. In that case firms and type-x individuals are located in region 1 while firms and type-y workers are located in region 2.

However this partial and total segregation are not the sole stable equilibria: between $\phi \in]\phi_l^d, \phi_l^b[$ and $]\phi_h^b, \phi_h^d[$ dispersion is also a possible equilibrium. Indeed if one worker moves, then the price of housing increases and thus the migrant prefers to return home, moreover if one firm deviates from this equilibrium, then local competition increases and the price index decreases, which reduces operating profit and then allows to stabilize the dispersed equilibrium.

Lastly between ϕ_l^b and ϕ_h^b agglomeration forces are sufficiently strong type x and type y individuals and firms are then agglomerated in one of the two regions.

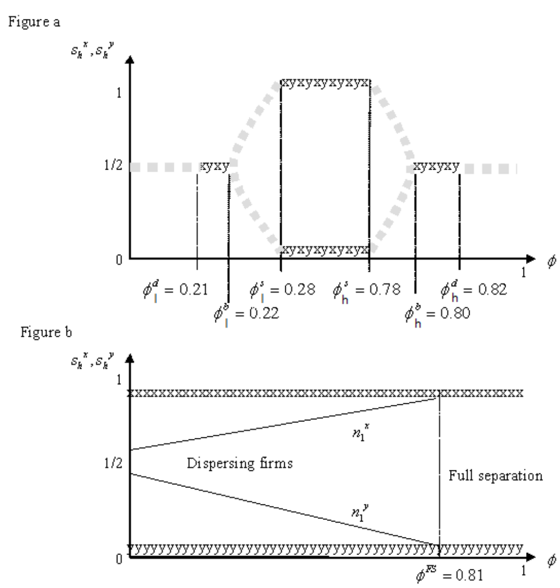

Figure 2. Bifurcation diagram

Those results predict the important role of path dependence in which even small shocks may give rise to large structural changes in the composition of one city.

A possible direction for future development would be to test empirically whether multiple equilibria are 'theoretical curiosities' or if an economy in which individuals are segregated can become an economy which mingles different people after a significant shock. [6]

4. Welfare Analysis

Until now we have only analyzed entrepreneurs' relative welfare. Here we propose a finer analysis by studying the individual welfare of the different interest groups which are given by:

$$V_i^{h^x} = w_i - \mu\eta \ln P_i^x - \mu(1-\eta)\ln P_i^y - \theta \ln p_{Hi} + \varepsilon$$
$$V_i^{h^y} = w_i - \mu\eta \ln P_i^y - \mu(1-\eta)\ln P_i^x - \theta \ln p_{Hi} + \varepsilon$$
$$V_i^L = 1 - \mu(\ln P_i^x + \ln P_i^y)/2 - \theta \ln p_{Hi} + \varepsilon$$

The objective is to examine these expressions under the three different equilibria (segregation, agglomeration and dispersion) in order to determine the best social outcome. Then we start this analysis with entrepreneurs' welfare and we find that:

Proposition 4: *If preference heterogeneity is relatively high and if preference for housing is sufficiently strong, then dispersion is the best outcome for mobile workers.*

Proof: *Under the different equilibrium (with subscript a for agglomeration, s for segregation and d for dispersion) the welfare of mobile workers is given by:*

$$V_a^{h^x} = [(1-\eta)\mu\rho \ln(2h-1) + (\rho-1)((h+2L)\mu - \theta\rho \ln(h(2+L)/H)]/(\rho-1)\rho$$
$$V_d^{h^x} = [-h\theta(\rho-1)\rho \ln((2h+L)/2H) + \mu((h+L)(\rho-1) + h\rho \ln(h(1+\phi)/2)]/h(\rho-1)\rho$$
$$V_s^{h^x} = (h+L)\mu / h\rho + \eta\mu \ln h/(\rho-1) - \theta \ln((2h+L)/2H) - (\eta-1)\mu \ln h\phi/(\rho-1)$$

Then the higher the differences between mobile workers, the more likely the dispersed equilibrium is going to be preferred in comparison with the segregated equilibrium. Indeed we find that:

$$V_s^{h^x} - V_d^{h^x} = \mu(\ln(2\phi/(1+\phi)) - \eta \ln \phi)/(\rho-1)$$

and $V_s^{h^x} - V_d^{h^x} < 0$ *if* $\eta > \ln((1+\phi)/2\phi)/\ln(1/\phi)$. *Concerning the agglomerated equilibrium and the dispersed equilibrium we find that* $V_s^{h^x} - V_a^{h^x} > 0$ *for the following critical value* θ^h *(the critical value of* ϕ *cannot be obtained analytically):*

$$\theta^h > [\mu(a + h(\eta-1)\rho \ln(2h-1) + h\rho \ln(h(1+\phi)/2)]/[h(\rho-1)\rho(\ln((2h+L)/2H) - \ln(h(2+L)/H))]$$
$$\text{With } a = (-h + h^2 - L + 2hL)(1-\rho)$$

Then in case of high taste heterogeneity and intense preference for housing we have $V_d^{h^x} > V_s^{h^x} > V_a^{h^x}$. *A similar reasoning can be carries out regarding type-y workers.* ***QED***.

[6] Exogeneous shocks such as wars or natural catastrophes (hurricane, tsunami etc) are usually used to analyse how historical accidents can shape the location of firms (Davis and Weinstein [3]) but to our knowledge these kinds of shocks have never been used to analyse the evolution of spatial segregation.

This result contrasts with a part of the literature which finds that mobile workers always prefer agglomeration to dispersion. It is also a complement of Pflüger and Suedekum [4] who demonstrate that dispersion can be preferred to agglomeration where an immobile housing stock is introduced, but who do not analyze the segregated equilibrium.

We can now analyze the situation of immobile workers by making a difference between those who are located in the potential Core (region 1) and those who are located in the potential Periphery (region 2). We find the following result:

Proposition 5: *If tastes heterogeneity is sufficiently strong and if the preference for housing is sufficiently weak, then dispersion is the best outcome for immobile workers.*

Proof: $V_d^{L_1} > V_s^{L_1}$ *if* $\eta < \eta^{L_1} \equiv \frac{\ln((1+\phi)/2\phi)}{\ln(1/\phi)}$ *and* $V_d^{L_1} > V_a^{L_1}$ *if:*

$$\theta > \theta^{L_1} \equiv \mu((\eta-1)\ln(2h-1)+\ln(h(1+\phi)/2)/[(\rho-1)(\ln((2h+L)/H)-\ln(h(2+L)/H)]$$

Then dispersion is preferred by immobile workers in region 1 when taste heterogeneity is sufficiently low and when preference for housing is sufficiently high. With respect to immobile workers who live in region 2 we get:

$$V_d^{L_2} > V_s^{L_2} \quad \textit{if} \quad \eta > \eta^{L_2} \equiv \ln(2/(1+\phi))/\ln(1/\phi) \quad \textit{and} \quad V_d^{L_2} > V_a^{L_2} \quad \textit{if:}$$

$$\theta < \theta^{L_2} \equiv \mu((\eta-1)\ln(2h-1)\phi+\ln(h(1+\phi)/2)-\eta\ln\phi)/[(\rho-1)(\ln((2h+L)/2H)-\ln(hL/H)]$$

Then dispersion is preferred by immobile workers in region 2 when taste heterogeneity is sufficiently strong and when preference for housing is sufficiently weak. ***QED****.*

The latter proposition linked to Proposition 4 raises an important question concerning the desirability of the dispersed equilibrium. Then thanks to the critical values of θ and η we find the following result.

Conjecture 1: *Dispersion can be Pareto improving.*

Proof: *Firstly Proposition 4 and 5 demonstrate that dispersion is preferred to segregation if* $\eta \in [\eta^{L_2}, \eta^{L_1}]$ *(note that* $\eta^{L_1} = \eta^h$ *). In Figure 3.a we plot* η^{L_2} *and* η^{L_1} *with respect to* ϕ*, which allows us to demonstrate that when* $\eta^{L_2} < \eta^{L_1}$ *, then it is possible to find a range of taste heterogeneity which verifies that* $\eta \in [\eta^{L_2}, \eta^{L_1}]$ *(the black area represents this situation). Proposition 4 and 5 indicate that dispersion is preferred to agglomeration if* $\theta > \theta^h$ *,* $\theta > \theta^{L_2}$ *and if* $\theta < \theta^{L_1}$*. In Figure 3.b*[7] *we plot* θ^h*,* θ^{L_2} *and* θ^{L_1} *with respect to* ϕ*, which allows us to see that we can find a range of housing preferences for which dispersion is preferred to agglomeration by everybody (again black area) .* ***QED****.*

[7]Parameters used in Figure 3.b are $h = L = H = 1$, $\eta = 0.501$, $\rho = 5$, $\mu = 0.6$. Figure 3.a is more general because η^{L_2} and η^{L_1} only depends on ϕ.

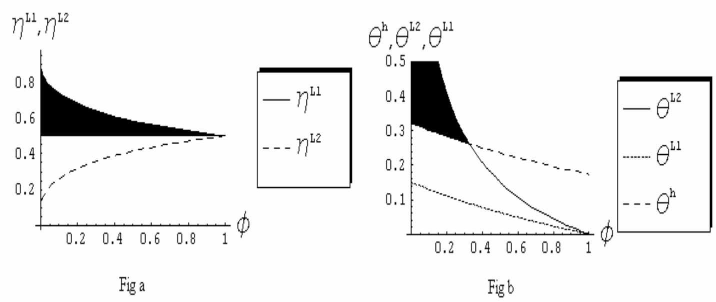

Figure 3. Pareto improvement

The proof of this result indicates that the Pareto domination of the dispersed equilibrium is less likely when trade is liberalized in cases where individuals have very different tastes and a weak preference for housing.

5. Welfare Analysis

By focusing on taste heterogeneity for private goods in a New Economic Geography model, Zeng [13] provided a new framework for the study of spatial segregation. His approach emphasizes that 1) increasing returns, trade liberalization and monopolistic competition lead to an endogenous persistent residential segregation, 2) dispersion is not a stable equilibrium 3) agglomeration is a better outcome than segregation except for unskilled workers in the peripheral region.

By introducing a housing stock we have checked the robustness of the result (1) but we have also found the vulnerability of conclusion (2) and (3) which depends heavily on the dispersive forces which are at work. In our model dispersion is a stable equilibrium because on the one hand firms' agglomeration exacerbates local competition while on the other hand workers' agglomeration increases housing prices. These higher prices in the Core obviously impact on individuals' well-being, and then we have shown that the dispersed equilibrium can be Pareto improving for that reason.

Obviously the present work is still in the early stages, the urban pattern needs to be modeled in greater details, for instance difference in lot size and commuting costs are natural parameters to improve the analysis. It may be particularly interesting to analyse how the different types of individuals choose their location among cities but also within each city. Lastly another important issue is to understand how urban sprawl impacts on spatial segregation.

Appendix: symmetric and mixed equilibrium

From the market clearing condition, we find nominal wages:

$$w_1 = \frac{\mu h}{\rho}\left(\frac{\eta s_{h^x} + (1-\eta)s_{h^y}}{n_1^x + \phi_t n_2^x} + \frac{(\eta(1-s_{h^x}) + (1-\eta)(1-s_{h^y}))\phi_t}{\phi_t n_1^x + n_2^x}\right),$$

$$w_1 = \frac{\mu h}{\rho}\left(\frac{\eta s_{h^y} + (1-\eta)s_{h^x}}{n_1^y + \phi_t n_2^y} + \frac{(\eta(1-s_{h^y}) + (1-\eta)(1-s_{h^x}))\phi_t}{\phi_t n_1^y + n_2^y}\right),$$

$$w_2 = \frac{\mu h}{\rho}\left(\frac{(\eta(1-s_{h^y}) + (1-\eta)(1-s_{h^x}))}{\phi_t n_1^y + n_2^y} + \frac{(\eta s_{h^x} + (1-\eta)s_{h^y})\phi_t}{n_1^y + \phi_t n_2^y}\right)$$

$$w_2 = \frac{\mu h}{\rho}\left(\frac{(\eta(1-s_{h^x}) + (1-\eta)(1-s_{h^y}))}{\phi_t n_1^x + n_2^x} + \frac{(\eta s_{h^x} + (1-\eta)s_{h^y})\phi_t}{n_1^x + \phi_t n_2^x}\right)$$

which allows to obtain w_1, w_2, n_1^x and N^x given in the text, and next by deriving these equations with respect to s_{h^x} we obtain a system of four equations with four unknowns. Then by resolving this system at the symmetric equilibrium $s_{h^x} = s_{h^y} = 0.5$ we get:

$$\frac{\partial w_1}{\partial s_{h^x}} = -\frac{\partial w_2}{\partial s_{h^x}} = \frac{\mu(1-\phi)(L(\phi-1)+2h\phi)}{h\rho(1+\phi)^2}$$

$$\frac{\partial n_1^x}{\partial s_{h^x}} = \frac{h(L(1-\phi)+2h(\eta(1+\phi)-\phi)}{2(h+L)(1-\phi)}$$

$$\frac{\partial N^x}{\partial s_{h^x}} = 0$$

We also calculate the derivatives of w_1, w_2, n_1^x, and N^x with s_{h^y} respect to which at the symmetric equilibrium are given by:

$$\frac{\partial w_1}{\partial s_{h^y}} = \frac{\partial w_1}{\partial s_{h^x}},\ \frac{\partial w_2}{\partial s_{h^y}} = -\frac{\partial w_1}{\partial s_{h^x}}$$

$$\frac{\partial n_1^y}{\partial s_{h^y}} = \frac{\partial n_1^x}{\partial s_{h^x}},\ \frac{\partial n_2^y}{\partial s_{h^y}} = -\frac{\partial n_1^x}{\partial s_{h^x}}$$

$$\frac{\partial n_1^x}{\partial s_{h^y}} = h - \frac{\partial n_1^x}{\partial s_{h^x}},\ \frac{\partial n_2^x}{\partial s_{h^y}} = \frac{\partial n_1^x}{\partial s_{h^x}} - h$$

Lastly concerning the derivatives of price indices we get:

$$\frac{\partial \Delta_1^x}{\partial s_{h^x}} = \frac{\partial \Delta_1^y}{\partial s_{h^y}} = \phi\frac{\partial N^x}{\partial s_{h^x}} + \frac{\partial n_1^x}{\partial s_{h^x}}(1-\phi)$$

$$\frac{\partial \Delta_2^x}{\partial s_{h^x}} = \frac{\partial \Delta_2^y}{\partial s_{h^y}} = \frac{\partial N^x}{\partial s_{h^x}} - \frac{\partial n_1^x}{\partial s_{h^x}}(1-\phi)$$

Thanks to these equations we determine:

$$\frac{\partial V^x}{\partial s_{h^x}} = \frac{4h^2L(-\theta c\rho b+\mu(1-5\phi+4\phi^2+\rho(1+6\phi-5\phi^2+d)))}{h(h+L)(2h+L)c\rho b}$$
$$-\frac{2L^3\mu ca^2-2hL^2\mu a(3-5\phi+\rho(-2+6\phi))+4h^3(\theta c\rho b-2\mu(a\phi+\rho(1+2\phi-\phi^2+d)))}{h(h+L)(2h+L)c\rho b}$$

$$\frac{\partial V^y}{\partial s_{h^x}} = \frac{-4h^2L(\theta c\rho b+\mu(-1+(5-4\rho)\phi+(-4+6\rho)\phi^2-\rho d))}{h(h+L)(2h+L)c\rho b}$$
$$-\frac{2L^3\mu ca^2-2hL^2\mu a(3-5\phi+\rho(-2+6\phi))-4h^3(\theta c\rho b+2\mu(\rho d+\phi(1+(-1+2\rho))))}{h(h+L)(2h+L)c\rho b}$$

with $a=\phi-1$, $b=(1+\phi)^2$, $c=\rho-1$, $d=-2\eta b+2\eta^2 b$. Lastly we find that:

$$\frac{\partial V^x}{\partial s_{h^y}} = \frac{\partial V^y}{\partial s_{h^x}}$$

$$\frac{\partial V^y}{\partial s_{h^y}} = \frac{\partial V^x}{\partial s_{h^x}}$$

Then the symmetric equilibrium is stable if

$$\frac{\partial V^x}{\partial s_{h^x}} < 0$$

and

$$\begin{bmatrix} \frac{\partial V^x}{\partial s_{h^x}} & \frac{\partial V^y}{\partial s_{h^y}} \\ \frac{\partial V^x}{\partial s_{h^y}} & \frac{\partial V^y}{\partial s_{h^x}} \end{bmatrix} = \frac{\begin{array}{c} -16(1-2\eta)^2 \mu(L^2 \mu c a^2 \\ + hL\mu a(2-4\phi+\rho(5\phi-1)) \\ + 2h^2(\theta c \rho b + \mu a(\rho - 2\phi + 3\rho\phi)))) \end{array}}{(h+L)(2h+L)c^2 \rho b} > 0$$

The first inequality is verified if $\phi \in [0, \phi_l^{\,b}[$ and/or if $\phi \in]\phi_h^{\,b}, 1]$ while the second one is verified for $\phi \in]\phi_l^{\,d}, \phi_h^{\,d}[$, where these critical values are given by:

$$\phi_l^{\,b} = \frac{\begin{array}{c} 2L^2\mu c + h^2 c(\mu - \theta\rho) + h3L\mu c \\ -\sqrt{((h+L)\mu L(\mu(1-2\rho)^2 - 8\theta c^2 \rho) + h(\mu(1-2\rho)^2 - 4\theta c^2 \rho))}) \end{array}}{2L^2 \mu c + hL\mu(-4+5\rho) + h^2(\theta c\rho + \mu(-2+3\rho)))}$$

$$\phi_h^{\,b} = \frac{\begin{array}{c} 2L^2\mu c + h^2 c(\mu - \theta\rho) + h3L\mu c \\ +\sqrt{((h+L)\mu L(\mu(1-2\rho)^2 - 8\theta c^2 \rho) + h(\mu(1-2\rho)^2 - 4\theta c^2 \rho))}) \end{array}}{2L^2 \mu c + hL\mu(-4+5\rho) + h^2(\theta c\rho + \mu(-2+3\rho)))}$$

And

$$\phi_l^{\,d} = \frac{\begin{array}{c} -4L^3\mu c + h^3(\theta c\rho + \mu(1+(-2+4\eta-4\eta^2)\rho)) \\ + h^2(L(2\theta c\rho + \mu(5+(-6+4\eta-4\eta^2)\rho)) + f) \\ + 2h(-4L^2\mu c + Lf) \end{array}}{\begin{array}{c} -4L^3\mu c - 2hL^2\mu(-5+6\rho) + 2h^2 L(-\theta c\rho + \mu(4+(-5-2\eta+2\eta^2)\rho)) \\ + h^3(-\theta c\rho + \mu(2+(-2+4\eta-4\eta^2)\rho)) \end{array}}$$

$$\phi_h^{\,d} = \frac{\begin{array}{c} 4L^3\mu c - h^3(\theta c\rho + \mu(1+(-2+4\eta-4\eta^2)\rho)) \\ - h^2(L(2\theta c\rho + \mu(5+(-6+4\eta-4\eta^2)\rho)) - f) \\ - 2h(4L^2\mu c - Lf) \end{array}}{\begin{array}{c} 4L^3\mu c + 2hL^2\mu(-5+6\rho) - 2h^2 L(-\theta c\rho + \mu(4+(-5-2\eta+2\eta^2)\rho)) \\ + h^3(\theta c\rho + \mu(-2+(2+4\eta-4\eta^2)\rho)) \end{array}}$$

with

$e = \mu(1-8(1-2\eta+2\eta^2)\rho + 8(1-2\eta+2\eta^2)\rho^2)$, $f = \sqrt{(h+L)\mu(L(-8\theta c^2\rho + e) + h(-4\theta c^2 + e))}$.

Although these critical values appear to be complicated, they become simpler at $\eta = 0.5$ where we get $\phi_l^{\,d} = \phi_l^{\,b}$ and $\phi_h^{\,d} = \phi_h^{\,b}$. This result is interesting because on the one hand $\phi_l^{\,d}$ and $\phi_h^{\,d}$ respectively decrease and increase with respect to η (see Table A) while $\phi_l^{\,b}$ and $\phi_h^{\,b}$ are independent of preference heterogeneity. As a result for $\eta > 0.5$, we always get $\phi_l^{\,b} > \phi_l^{\,d}$ and $\phi_h^{\,d} > \phi_h^{\,b}$, thus inequalities previously presented hold when $\phi \in]\phi_l^{\,d}, \phi_l^{\,b}[$ and when $\phi \in]\phi_h^{\,b}, \phi_h^{\,d}[$, which guarantees the stability of the dispersed equilibrium.

Table 1. Critical points of trade opening

	$\eta = 0.5$	$\eta = 0.6$	$\eta = 0.75$	$\eta = 0.95$
ϕ_l^b	0.218	0.218	0.218	0.218
ϕ_l^d	0.218	0.211	0.176	0.127
ϕ_h^b	0.805	0.805	0.805	0.805
ϕ_h^d	0.805	0.821	0.905	1.11323

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